Concept Building Assignment
CONVEX CLASSES
Relation and Function

Exercise 1.1 [NCERT]

(1) Determine whether each of the following relations are reflexive, symmetric and transitive.

(i) Relation R in the set $A=\{1,2,3,...13,14\}$ defined as $R=\{(x,y):3x-y=0\}$

Sol. Reflexive : $\because\ 3x-y \Rightarrow y=3x$,

put $x=1,\ y=3,\ \Rightarrow (1,3)$

Put $x=2,\ y=3\,(2)=6 \Rightarrow (2,6)$,

put $x=3,\ y=9 \Rightarrow (3,9)$

Put $x=4,\ y=12 \Rightarrow (4,12)$

so, $R=\{(1,3)\ (2,6)\ (3,9)\ (4,12)\}$

This relation is not reflexive. $\because (1,1)\notin R$

Symmetric : $R^{-1}=\{(3,1),(6,2)(9,3)....(12,4)\}$ $\quad \because R\neq R^{-1}$

so relation it is not symmetric relation.

Transitive : $(1,3)\in R,\ (3,9)\in R \Rightarrow (1,9)\notin R$

hence given relation is not Transitive. **Ans**

(ii) Relation R in the set **N** of a natural number defined as $R=\{(x,y): y=x+5, x<4\}, R\in N\times N$

[Kashmir 2012]

Sol. $\because\ y=x+5$ and $x<4$,

put $x=1, y=1+5=6 \Rightarrow (1,6)$

put $x=2, y=7 \Rightarrow (2,7)$,

put $x=3,\ y=8 \Rightarrow (3,8)$

hence $R=\{(1,6)\ (2,7)\ (3,8)\}$

Reflexive: $(1,1)\notin R$ so given relation is not reflexive relation.

Symmetric : $R^{-1}=\{(6,1)\ (7,2)\ (8,3)\}$

$\because\ R\neq R^{-1}$, given relation is not Symmetric.

Transitive : $x\,R\,y \Rightarrow\ y=x+5$ ....(1)

and $y\,R\,z \Rightarrow\ z=y+5$............(2)

from equation (1) and (2), $\quad z=(x+5)+5$

$$z=x+10\neq x+5 \not\Rightarrow x \not R Z$$

Hence given relation is not transitive relation. **Ans**

(iii) $A=\{1,2,3,4,5,6\}$, $R=\{(x,y): y$ is divisible by x $x,y\in A\}$

$R=\{(1,2)(1,3)(1,4)(1,5)(1,6)(2,4)(2,6)(3,6)(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$

$R^{-1}=\{(2,1)(3,1)(4,1)(5,1)(6,1)(4,2)(6,2)(6,3)(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$

Reflexive: Each element is related with itself , hence it is reflexive relation.

Symmetric : $\because\ R\neq R^{-1}$,hence given relation is not symmetric.

Transitive: $(x,y)\in R,(y,z)\in R \Rightarrow (x,z)\in R$, true for each element.

hence it is transitive relation . **Ans**

(iv) R is defined on the set of Integer Z

R={(x,y) : x - y is an Integer $x,y\in Z$ }.

[AIEEE 2008, Punjab B. 2011]

Sol. Reflexive : $x\ R\ x=x-x=0\in Z$ (Integer),

hence it is reflexive relation.

Symmetric : $x\,R\,y=x-y$ is integer

$x\ R\ y \Rightarrow x-y=I_1$ (Let)............(1), multiply (-1) both side $\quad y-x=-I_1$ (also Integer) $\Rightarrow y\,R\,x$,

hence it is also Symmetric relation.

Transitive : $x\,R\,y=x-y=$ Integer (I_1) (Let).............(1)

$y\,R\,z=y-z=$ Integer (I_2) (Let)...............(2)

equation (1) + (2) , $\quad x-y+y-z=I_1+I_2$

Sum of two integer is also integer, hence

$x-z=I_3\in Z \Rightarrow x\,R\,z$, hence it is also transitive. **Ans**

(v) Relation R in the set A of human being in a town at a particular time given by

(a) R ={(x,y) : x and y work at the same place}

Reflexive : $x\ R\ x \Rightarrow$ x and x work at the same place ,

it is always true.

hence it is reflexive relation.

Symmetric : $x\ R\ y \Rightarrow$ x and y work at the same place

$\Rightarrow$ y and x will also be work at the same place $\Rightarrow y\ R\ x$,

hence it is symmetric relation.

Transitive : $x\ R\ y \Rightarrow$ x, y work at the same place

$y\ R\ z \Rightarrow$ y, z work at the same place

then x and z will also be work at the same place

$\Rightarrow x\ R\ z$ means this relation is also transitive

since the given relation is reflexive, symmetric , transitive hence it equivalence relation. **Ans**

(b) R = {(x,y) : x and y live in the same locality}

Sol. Reflexive : $x\ R\ x \Rightarrow$ x and x live in the same locality. Thus it is always true.

hence given relation is reflexive relation.

Symmetric : $x\ R\ y \Rightarrow$ x and y live in the same locality.

then y and x will also be live in the same locality $\Rightarrow y\ R\ x$

Hence it is symmetric relation.

Transitive : $\Rightarrow y\ R\ x$ x, y live in the same locality.

$y\ R\ z \Rightarrow$ y, z live in the same locality.

$\Rightarrow$ then x and z will also be live in the same locality

$\Rightarrow x\ R\ z$, Hence it is transitive relation. **Ans**

(c) R ={ (x,y): x is exactly 7cm taller then y }

Reflexive : $x\ R\ x \Rightarrow$ x is 7cm taller then x , it is not possible.

Any person can't be taller then himself mean relation is not reflexive $\Rightarrow x \not{R}\ x$

Symmetric : $x\ R\ y =$ x is 7 cm taller then y.

$\Rightarrow$ y cannot be 7 cm taller then x .

$\Rightarrow y \not{R}\ x$,Hence the relation is not Symmetric.

Transitive : $x\ R\ y \Rightarrow$ x is 7 cm taller then y...............(1)

and $y\ R\ z \Rightarrow$ y is 7 cm taller then z ....(2)

then x will be 14 cm taller then z . Hence x is not exactly 7 cm taller then $z \Rightarrow x \not{R}\ z$ given relation is not transitive.

(d) R = {(x,y) : x is wife of y}

Sol Reflexive : $x\ R\ x \Rightarrow$ x is the wife of x not possible.

Hence the relation is not reflexive.

Symmetric $x\ R\ y \Rightarrow$ x is the wife of y then y can't be the wife of x . $\Rightarrow$ y will be the husband of x $\Rightarrow$ Hence $y \not{R}\ x \Rightarrow$ Relation is not Symmetric.

Transitive: $x\ R\ y \Rightarrow$ x is wife of y

$y\ R\ z \Rightarrow$ y is the wife of z ,both these relation never true.

hence relation is not transitive. **Ans**

(e) R = {(x,y) : x is the father of y}

Sol. Reflexive : $x\ R\ x \Rightarrow x$ is father of x.

any person can't be father of himself hence given relation is not reflexive.

Symmetric : $x\ R\ y \Rightarrow$ x is the father of y then y can't be the father of $x \Rightarrow y \not{R}\ x \Rightarrow$ not symmetric.

Transitive : $x\ R\ y \Rightarrow$ x is the father of y...........(1)

and $y\ R\ z \Rightarrow$ y is the father of z ....(2)

then x will be the grand father of z

Hence x can't be the father of z. $\Rightarrow x \not{R}\ z$

Given relation is not transitive. **Ans**

(2) Show that the relation R on the set of real numbers, defined as $R = \{(a,b) : a \le b^2\}$ is neither reflexive nor Symmetric nor transitive.

[RBSE 2014 ; CBSE 2009 (F), Punjab B. 2013]

Sol. Reflexive: $a \not\le a^2 \Rightarrow a \not{R}\ a$, given relation is not reflexive.

Symmetric: $a\ R\ b \Rightarrow a \le b^2$...........(1) and

$b\ R\ a \Rightarrow b^2 \le a$ ....(2)

Both equation never possible.

hence the given relation is not Symmetric.

Transitive: $aRb \Rightarrow a \le b^2$ ....(1)

and $bRc \Rightarrow b \le c^2$ ....(2)

from equation (1) and (2) , $a \le (c^2)^2 \Rightarrow a \le c^4$

$\Rightarrow a \not\le c^2 \Rightarrow a \not{R}\ c$, hence relation is not transitive. **Ans**

(3) Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a,b) : b = a + 1} is reflexive, symmetric or transitive. [CBSE 2007 (AI)]

Sol. Reflexive: $a\ R\ a \Rightarrow a = a + 1$, it is not possible

$\Rightarrow a \not{R} a \Rightarrow$ given relation is not reflexive.

Symmetric: $aRb \Rightarrow b = a+1$......(1) , $bRa \Rightarrow a = b+1$.........(2)

from equation (1) and (2) , $a = (a+1)+1$

$a = a+2 \Rightarrow 2 = 0$, not possible

$a\ R\ a \Rightarrow b \not{R}\ a$ relation is not Symmetric.

Transitive: $a\ R\ b \Rightarrow b = a+1$.........(1) & $bRc \Rightarrow c = b+1$ ....(2)

From equation (1) and (2)

$c = (a+1)+1 = a+2 \Rightarrow c \ne a+1 \not\Rightarrow a \not{R}\ c$

hence given relation is not transitive.

Hence given relation is neither reflexive nor symmetric nor transitive.

(4) Show that the relation R in R defined as $R = \{(a,b) : a \le b\}$ is reflexive and transitive but not symmetric.

[CBSE 2003(Delhi) , 2001(Delhi) , 2001(AI), H.B 2013, Kashmir 2011]

Sol. Reflexive: $a\ R\ a \Rightarrow a \le a \Rightarrow$ true when $a = a$

hence , given relations is reflexive relation.

Symmetric: $a\ R\ b \Rightarrow a \le b$.........(1) & $b\ R\ a \Rightarrow b \le a$ ....(2)

both these relation will be satisfied only when a = b.

Hence it is skew symmetric relation

Transitive: $a\ R\ b \Rightarrow a \le b$ ....(1)

& $b\ R\ c \Rightarrow b \le c$ ....(2)

from equation (1) and (2) , $a \le b \le c$

$a \le c \Rightarrow a\ R\ c$, hence the relation is transitive.

$\because$ Relation is reflexive, skew symmetric and transitive .

hence it is **partial order relation** . **Ans**

(5) Check whether the Relation R in R defined by $R=\{(a,b): a\le b^3\}$ is reflexive, symmetric or transitive. [CBSE 2010, Jammu 2013]

Sol. $aRa \Rightarrow a \le a^3$ (not possible except a = 1)

Hence relation is not reflexive $\Rightarrow a \not R\, a$

Symmetric : $aRb \Rightarrow a \le b^3$ ....(1) &

$bRa \Rightarrow b \le a^3$ ......(2)

both these relation not satisfied

hence $a\,R\,b \not\Rightarrow b\,R\,a$, so given relation is not Symmetric.

Transitive : $aRb \Rightarrow a \le b^3$ ....(1) and

$bRc \Rightarrow b \le c^3$ ....(2)

from equation (1) and (2) , $a \le (c^3)^3$

$a \le c^9 \Rightarrow a \not\le c^3 \Rightarrow a \not R\, c$, so it is not transitive.

since given relation is neither reflexive, nor symmetric and not transitive

(6) Show that the relation R in the set {1, 2, 3} given by $R=\{(1,2)\ (2,1)\}$ is symmetric but neither reflexive nor transitive.
[CBSE 2004(Delhi),2003(AI) , 2011]

Sol. Reflexive: $(1,1)\notin R$,hence R is not reflexive relation.

Symmetric : $R^{-1}=\{(2,1)\ (1,2)\} \Rightarrow R=R^{-1}$,

hence R is symmetric relation.

Transitive : $(1,2)\in R, (2,1)\in R \Rightarrow (1,1)\notin R$

Hence the relation is not Transitive.

since the given relation is not reflexive , symmetric and not transitive.

(7) Show that the relation R in the set A of all the books in a library of a college, given by R = {(x,y) : x and y have same number of pages} is an equivalence relation.

Sol. Reflexive: $xRx \Rightarrow x$ and x have same number of $\Rightarrow$ pages true for each x. hence relation is reflexive.

Symmetric : $xRy \Rightarrow x$ and y have same number of page $\Rightarrow$ y and x have same no of page $\Rightarrow yRx$

hence , given relation is symmetric relation.

Transitive : $xRy \Rightarrow x\ \&\ y$ have same number of page,

$y\ R\ z \Rightarrow$ y and z have same number of page .

Then x and z will also have same number of page

$\Rightarrow x\ R\ z \Rightarrow$ transitive relation .

$\because$ R is reflexive, symmetric and transitive .

hence it is equivalence relation. **Ans**

(8) Show that the relation R in the set A = {1, 2, 3, 4, 5} given by $R=\{(a,b): |a-b|$ is even} is an equivalence relation. Show that all the elements of {1, 3, 5} are related each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}. [CBSE 2009(Delhi)]

Sol. Reflexive: $aRa=|a-a|=0 \Rightarrow$ even integer .

hence it is reflexive relation.

Symmetric : $aRb \Rightarrow |a-b|=$ even integer $=2k\ \{k\in Z(let)\}$

$\Rightarrow |b-a|=|-(a-b)|=|-2k|=2k$ even integer $\Rightarrow bRa$

hence given relation is symmetric relation.

2nd method : $R=\{(1,3)(1,5)(2,4)(3,5)\ (3,1)\ (5,1)\ (4,2)\ (5,3)\}$

$R^{-1}=\{(3,1)(5,1)(4,2)(5,3)(1,3)(1,5)(2,4)(3,5)\}$

$\because R=R^{-1}$, hence given relation is **symmetric**.

Transitive: $aRb=|a-b|=$ even integer $=2k_1$ ....(1)

$bRc=|b-c|=$ even integer $=2k_2$ ....(2)

$|a-c|=|a-b+b-c|=|a-b|+|b-c|=2k_1+2k_2$

$|a-c|=2(k_1+k_2)=$ even integer $\Rightarrow aRc$

hence the given relation is transitive. It is reflexive, symmetric, transitive relation .hence it is **equivalence** relation . **Ans**

(9) Show that each of the relation R in the set $A=\{x\in Z: 0\le x\le 12\}$, given by

(i) $R=\{(a,b): |a-b|$ is a multiple of 4$\}$ **[CBSE 2010 (AI)]**

(ii) $R=\{(a,b): a=b\}$

Sol . Set $A=\{x\in Z: 0\le x\le 12\}=\{0,1,2,........,12\}$,

(i) $a\ R\ b \Rightarrow |a-b|$ will be multiple of 4 when '**a**' and '**b**' are the elements of set {1,5,9} . hence $R=\{(1,1)\ (5,5)\ (9,9)\ (1,5)\ (1,9)\ (5,1)\ (9,1)\ (5,9)\ (9,5)\}$

Reflexive : Each element of this set is related with itself .

hence it is **reflexive relation or**

$a-a=0=0\times 4$, multiple of 4 $\Rightarrow (a,a)\in R$

hence it is **reflexive relation .**

Symmetric : $R=R^{-1}$, hence R is symmetric.

Transitive : $aRb=|a-b|$ multiple of $4=4k_1$ (Let) ....(1)

$bRc=|b-c|$ multiple of $4=4k_2$ (Let) ....(2)

$|a-c|=|a-b+b-c|=4(k_1+k_2)$, multiple of 4 $\Rightarrow aRc$

given relation is transitive relation .

Since the given relation is reflexive, symmetric, transitive hence it is **equivalence** relation . **Ans**

Sol(ii) $R=\{(a,b): a=b\}$

$a=a \Rightarrow$ R is reflexive, $a=b \Rightarrow b=a$, R is symmetric

$a=b,\ b=c \Rightarrow a=c$ R is transitive

Hence R is equivalence relation **Ans**

(10) Given an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor Symmetric .
[RBSE 1999]

(iii) Reflexive and Symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.
(v) Symmetric and transitive but not reflexive.

Sol(i) solution see on Important example (9)

Sol(ii) If relation R is defined on set of real number such that $x R y \Rightarrow x > y$.

Reflexive :

$x R x \Rightarrow x \not> x$,hence given relation is not reflexive.

Symmetric : $x R y \Rightarrow x > y \not\Rightarrow y \not R x$, not symmetric .

Transitive : $x R y \Rightarrow x > y,\ \ y R z \Rightarrow x > z$

$x > y > z \Rightarrow x > z , x R z$, hence given relation is transitive.

since the given relation is transitive but neither reflexive nor symmetric .

Sol(iii) $a R b \Leftrightarrow 1 + ab > 0\ \forall\ a, b \in R$

solution see on Important example (5)

Sol(iv) Solution see on Important example (2)

Sol(v) Reflexive: $x\ R\ x\ \Rightarrow$ '**x**' is the brother of '**x**' not true , any person can't be the brother of himself , hence given relation is not reflexive.

Symmetric : $x\ R\ y\ \Rightarrow$ x is the brother of y then y will also be the brother of $x \Rightarrow y R x$

Transitive : $x\ R\ y \Rightarrow$ x is the brother of '**y**'

$y\ R\ z \Rightarrow$ '**y**' is the brother of '**z**' then '**x**' will also be the brother of '**z**' $\Rightarrow x\ R\ z$ transitive.

since the given relation is symmetric and transitive but not reflexive.

(11) Show that the relation R in the set A of points in a plane given by R = {(P,Q) : distance of point P from origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

Sol. Reflexive : $PRP \Rightarrow OP = OP$ true, hence the given relation is reflexive.

Symmetric : $PRQ \Rightarrow OP = OQ$

$OQ = OP \Rightarrow QRP$

Transitive: $P R Q \Rightarrow OP = OQ$ ....(1)

& $QRS \Rightarrow OQ = OS$ ....(2)

$OP = OQ = OS$, hence given relation is transitive.

since the given relation is reflexive, symmetric and transitive. hence it equivalence relation. **Ans**

(12) Show that the relation R defined in the set A of all triangle as $R = (T_i, T_2) : T_1$ is similar to T_2 }, is equivalence relation.

Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related ? [CBSE 2008]

Sol. Given relation is $T_1\ R\ T_2 \Rightarrow T_1$ is similar to T_2

Reflexive : $T_1\ R\ T_1 \Rightarrow$ each triangle is similar to itself, hence given relation is reflexive relation .

Symmetric : If T_1 triangle is similar to T_2 then T_2 will also be the similar to T_1.

hence the given relation is symmetric.

Transitive : $T_1 R T_2 \Rightarrow \Delta T_1 \approx \Delta T_2$ ....(1)

and $T_2 R T_3 \Rightarrow \Delta T_2 \approx \Delta T_3$ ....(2)

From equation (1) and (2) ,

$\Delta T_1 \approx \Delta T_2 \approx \Delta T_3 \Rightarrow \Delta T_1 \approx \Delta T_3 \Rightarrow T_1 R T_3$

hence given relation is transitive relation. since the given relation is reflexive, symmetric and transitive. so it is called equivalence relation

(13) Show that the relation R defined in the set A of all polygons as $R = \{(P_1,\ P_2) : P_1$ and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5 ?

Sol. Reflexive: $P_1\ R\ P_1 \Rightarrow P_1$ and P_1 have same number of sides it is true for all triangle. hence it is reflexive relation.

Symmetric: $P_1\ R\ P_2 \Rightarrow P_1$ & P_2 have same number of sides then P_2 and P_1 also having equal number of side $\Rightarrow P_2\ R\ P_1$, so it is symmetric relation.

Transitive: $P_1\ R\ P_2 \Rightarrow P_1$ & P_2 has equal side.

Let $P_2\ R\ P_3 \Rightarrow P_2$ & P_3, also having equal side then

$\Rightarrow P_1$ & P_3 will also be equal number of side. $\Rightarrow P_1\ R\ P_3$,

hence given relation is transitive . since the given relation is reflexive, symmetric and transitive . so it is called equivalence relation. **Ans**

(14) Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1,\ L_2) : L_1$ is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$. [CBSE 2010]

Sol. Reflexive: $L_1 R L_1 \Rightarrow L_1 \parallel L_1$ each line is parallel to itself . hence given relation is reflexive.

Symmetric : $L_1 R L_2 \Rightarrow L_1 \parallel L_2 \Rightarrow L_2 \parallel L_1$

hence given relation is symmetric relation.

Transitive: $L_1 R L_2 \Rightarrow L_1 \parallel L_2$........(1)

and $L_2 R L_3 \Rightarrow L_2 \parallel L_3$ ......(2)

$L_1 \parallel L_2 \parallel L_3 \Rightarrow L_1 R L_3$ relation is transitive.

since given relation is reflexive , symmetric and transitive , hence it is called equivalence relation. **Ans**

(15) Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.
(B) R is reflexive and transitive but not symmetric.
(C) R is symmetric and transitive but not reflexive.
(D) R is an equivalence relation.

Sol. Reflexive : $\{(1,1)(2,2)(3,3)(4,4)\} \in R$ each element of this set is related to itself hence it is reflexive relation.

Symmetric : $(1,2) \in R, (2,1) \notin R$, $\therefore$ R is not symmetric.

Transitive : $(1,3) \in R, (3,2) \in R \Rightarrow (1,2) \in R$

hence **R** is transitive, relation R is reflexive, transitive but not symmetric . So it is **partical order** relation.

Ans [B]

(16) Let R be the relation in the set N given by $R = \{(a,b) : a = b-2 , b > 6\}$

choose the correct answer.

(A) $(2,4) \in R$ (B) $(3,8) \in R$ (C) $(6,8) \in R$

(D) $(8,7) \in R$

Sol. Given $aRb \Rightarrow a = b-2, b > 6$

Let b= 6 , $a = b-2 = 6-2 = 4 \Rightarrow (4,6)$

Let b= 7 , $a = b-2 = 7-2 = 5 \Rightarrow (5,7)$

Let b= 8 , $a = b-2 = 8-2 = 6 \Rightarrow (6,8)$

$R = \{(4,6)(5,7)(6,8),(7,9)(8,10).......\} \ \forall a, b \in N$

Ans [C]

Exercise 1.2 [NCERT]

(1) Show that the function $f : R_* \to R_*$ **defined by** $f(x) = 1/x$ **is one-one and onto, where** R_* **is the set of all non-zero real numbers. Is the result true, if the domain** R_* **is replaced by N with co-domain being same as** R_* **? [CBSE 2009 (AI)]**

Sol. One-One/Many-one: $x, y \in R_0$ such that

$f(x) = f(y)$ then $\Rightarrow 1/x = 1/y \Rightarrow x = y$

So, function is one - one.

Onto/ into : $y = 1/x \Rightarrow x = 1/y$

This function is defined for each y except y = 0

hence range = R_0 = Co-domain

hence, function is onto.

Now , $f : N \to R$, $f(x) = 1/x$

Range = $f(N) = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}......\right\} \neq$ co-domain

hence function is into.

(2) Check the injectivity and surjectivity of the following functions :

(i) $f : N \to N$, $f(x) = x^2$ **[CBSE 2004 (AI)]**

Sol. Given $f : N \to N$ and $f(x) = x^2$

where N = {1, 2, 3, 4, ...} = set of natural numbers.

One-One/Many-one: Let $a, b \in N$ such that

$\therefore \ f(a) = f(b) \Rightarrow a^2 = b^2 \Rightarrow b = \pm a$

$b = -a$ not possible , hence $b = a$ is only solution

so, $f(a) = f(b) \Rightarrow a = b \ \forall a, b \in N$

so, f is one-one mapping.

Onto / Into :

$x = 1, \ y = 1^2 = 1$, $x = 2, \ y = 2^2 = 4$, $x = 3, \ y = 3^2 = 9$

so $f(N) = \{1^2, 2^2, 3^2, 4^2, ...\}$ = Range of f

hence, range $\subset$ co-domain (function is into)

hence function is one-one into. **Ans**

(ii) $f : Z \to Z$ given by $f(x) = x^2$ **[CBSE 2005 (Delhi)]**

Sol.One-One/Many-one :

$f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = \pm y$

$y = x, -x$ two values of x ,

hence it is many -one mapping.

e.g., $x = 1, \ f(1) = 1^2 = 1$, $x = -1, \ f(-1) = (-1)^2 = 1$

$1 \neq -1$, $f(1) = f(-1)$, function is many one.

2nd method : $x \neq (-x) \Rightarrow (x)^2 = (-x)^2 \Rightarrow f(x) = f(-x)$

hence function is many-one.

Onto / Into : $x \in Z, x^2 \in Z^+ \Rightarrow y \in Z^+$

hence Range = $\{0, 1, 2, 3, ...\}$

co-domain $= \{...... -3, -2, -1, 0, 1, 2, 3, ...\}$

$\because$ Range $\subset$ co-domain

so function is into, hence function is many- one into .

Ans

(iii) $f : R \to R$, $f(x) = x^2$ **[IIT 1970; MPPET 1997]**

Sol. One-One/Many-one: $\because \ x \neq -x \Rightarrow x^2 = (-x)^2$

$f(x) = f(-x)$ function many-one .

Onto / Into: $x \in R$ so $x^2 \in R^+ \Rightarrow y \in R^+$ so, Range = R^+

Co-domain = R, $\therefore$ Range $\subset$ Co-domain So, function is into.

hence function is many -one into. **Ans**

(iv) $f : N \to N$ given by $f(x) = x^3$

Sol. One-One/Many-one : $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$

hence it is one one mapping

Onto / Into : Put $x = 1,\ f(1) = 1^3 = 1$,

Put $x = 2,\ f(2) = 2^3 = 8$

Put $x = 3,\ f(3) = 3^3 = 27$

hence Range $= \{1, 8, 27, ...\}$, co-domain $= \{1, 2, 3, ...\}$

$\because$ Range $\subset$ co-domain hence function is into mapping

So, function is one-one into. **Ans**

(v) $f : Z \to Z$ given by $f(x) = x^3$

Sol. One-One/Many-one : $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$

Hence function is one one .

Onto / Into : $f : Z \to Z$, $y = x^3$

$x = (y)^{1/3}$, $y \in Z \Rightarrow y^{1/3} \notin Z \Rightarrow x \notin Z$,

Function is into .

Hence given function is one-one into. **Ans**

(3) Prove that the Greatest Integer Function $f : R \to R$ given by $f(x) = [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x. [KCET 2004]

Sol. One-One/Manyone: $x \neq y \Rightarrow [x] = [y] \Rightarrow f(x) = f(y)$

$\because x \neq y \Rightarrow f(x) = f(y)$

e.g., $1/2 \neq 1/3 \Rightarrow f(1/2) = 0 = f(1/3)$

hence this function is many one mapping.

Onto / Into : $y = [x],\ x \in R$

$x \in R \Rightarrow [x] \in Z \Rightarrow y \in Z$, Range = Z

Co-domain = R, Range $\subset$ co-domain ,

function is into Hence function is many-one into.

Ans

(4) Show that the modulus Function $f : R \to R$, given by $f(x) = |x|$, is neither one-one nor onto, where |x| is x, if x is positive or 0 and |x| is -x , if x is negative.

Sol. One-One/Many-one: $x \neq -x \Rightarrow |x| = |-x|$

$\Rightarrow f(x) = f(-x)$, hence it is many one mapping

Onto / Into: $x \in R \Rightarrow |x| \in R_+$

Range = R_+ , co-domain = R

Range $\subset$ co-domain $\Rightarrow$ into mapping.

Hence function is many one into. **Ans**

(5) Show that the Signum Function $f : R \to R$, given by $f(x) = \begin{cases} 1\ , & x > 0 \\ 0\ , & x = 0 \\ -1\ , & x < 0 \end{cases}$ **is neither one-one nor onto.**

Sol. One-One/Many-one: Case I : If $x > 0$, Let $x_1, x_2 \in R^+$

$x_1 \neq x_2$, $f(x_1) = f(x_2) = 1$, hence function is many one

e.g., $1 \neq 2 \Rightarrow f(1) = f(2) = 1$

onto/into: Range $= \{-1, 0, 1\}$, co-domain = R

Range $\subset$ co-domain $\Rightarrow$ function is Into mapping,

hence function is many-one into. **Ans**

(6) Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one. [CBSE 2011 (AI)]

Sol. $f = \{(1, 4)\ (2, 5)\ (3, 6)\}$,

No second element repeated hence function is one-one . **Ans**

(7) In each of the following case, state whether the function is one-one, onto or bijective. Justify your answer.

(i) $f : R \to R$ defined by $f(x) = 3 - 4x$

[CBSE 2010 C (Similar)]

Sol. One-One/Many-one : Let $f(x) = f(y)$

$\Rightarrow 3 - 4x = 3 - 4y \Rightarrow x = y$ function is one one.

onto/ into : $y = 3 - 4x \Rightarrow x = (3 - y)/4$

$y \in R$, then $3 - y \in R \Rightarrow \dfrac{3 - y}{4} \in R \Rightarrow x \in R$

$\Rightarrow$ for each y there exist x in domain (R) so $f(x)$ is onto.

Hence function is one one onto. **Ans**

(ii) $f : R \to R$ defined by $f(x) = 1 + x^2$

[Similar CBSE 2005 (D.B)]

Sol. One-One/Many-one : $f(x) = f(y) \Rightarrow 1 + x^2 = 1 + y^2$

$\Rightarrow y = \pm x$, Two values for y , hence function is many one.

Onto/into : $y = 1 + x^2 \Rightarrow x = \sqrt{1 - y}$

x will be defined when $1 - y \geq 0 \Rightarrow y \leq 1$

Range $= (-\infty, 1]$, co-domain = R ,

Range $\subset$ co-domain $\Rightarrow$ function is into .

Hence it is many one into . **Ans**

(8) Let A and B be sets, Show that $f : A \times B \to B \times A$ such that f (a, b) = (b, a) is bijective function.

Sol. One-One/Many-one : Let (a_1, b_1) and $(a_2, b_2) \in A \times B$

Such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2) \Rightarrow b_1 = b_2$, $a_1 = a_2$

so, $(a_1, b_1) = (a_2, b_2)$ hence

$f(a_1, b_1) = f(a_2, b_2) \Rightarrow (a_1, b_1) = (a_2, b_2)$

Hence function is one one or injective mapping.

Onto/into : let (b, a)is an arbitrary element of $B \times A$

Then $b \in B$ and $a \in A$ hence $(a, b) \in (A \times B)$

then $(b,a) \in B \times A \Rightarrow f(a,b) = (b,a) \in B \times A$

hence range = $B \times A$ = co-domain. function is onto.

Given function is one-one onto (Bijection). **Ans**

(9) Let $f : N \to N$ be defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if n is odd} \\ n/2 & \text{it n is even} \end{cases}$$ **for all $n \in N$, State whether the function f is bijective. Justify your answer.**

[CBSE 2012-2009(AI); AIEEE 2009 ; KCET 2014]

Sol. One -one / many -one : $f(1) = \frac{1+1}{2} = 1$, $f(2) = \frac{2}{2} = 1$

thus $1 \neq 2 \Rightarrow f(1) = f(2)$. So, f is many one function.

Onto / into : $n \in$ odd, $n+1 \in$ even

$\frac{n+1}{2}$ = integer , $f(n) \in$ integer ,

If $n \in$ even, $n/2 \in$ integer, $f(n) \in$ integer

hence the range of the function is = integer =co-domain

So, f is surjective ,hence f is many one onto. **Ans**

(10) Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = (x-2)/(x-3)$. Is f one-one and onto ? Justify your answer.

[CBSE 2014 C (Delhi) ; 2012(Delhi) ,2006(Delhi); RBSE 2012]

Sol. One -one / many -one : $f(x) = f(y) \Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$

$\Rightarrow xy - 2y - 3x + 6 = xy - 2x - 3y + 6 \Rightarrow x = y$

$\because f(x) = f(y) \Rightarrow x = y$.

Hence f is one one mapping .

Onto/ into : $y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2$

$x(y-1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}$

This function is not defined for y = 1 only.

Hence Range = R – {1}= co-domain .

function is onto. Hence function is one one onto (bijective). **Ans**

(11) Let $f : R \to R$ be defined as $f(x) = x^4$.

Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

[CBSE 2006(AI)]

(C) f is one-one but not onto

(D) f is neither one-one nor onto.

Sol. One one/ many one : $x \neq -x \Rightarrow x^4 = (-x)^4$

$\Rightarrow f(x) = f(-x)$, Hence f is many one

Onto/ into : $x \in R$, $x^4 \in R_+$, $f(x) \in R_+$

Range = R_+, co -domain =R $\Rightarrow$ Range $\subset$ co-domain

Hence function is into. So given function is many-one into (bijective) **Ans**

(12) Let $f : R \to R$ be defined as $f(x) = 3x$. Choose the correct answer. **[Similar CBSE 2008 (AI); PB 2010]**

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto

(D) If is neither one-one nor onto.

Sol. One one/ many one: $f : R \to R$, $f(x) = 3x$

$f(x) = f(y) \Rightarrow 3x = 3y \Rightarrow x = y$, one-one mapping

Onto / into : y=3x $\Rightarrow$ x = $y/3$

$y \in R \Rightarrow \frac{y}{3} \in R \Rightarrow x \in R$,

$\because$ for each y there exist x in domain

hence function is onto. so function is one-one onto (bijection). **Ans**

Miscellaneous Exercise

(1) Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$, such that $gof = fog = 1_R$

[CBSE 2011(AI)]

Sol. $f(x) = 10x + 7$, Put $g(x)$ at the place of x

$f[g(x)] = 10g(x) + 7$ given $fog(x) = x$

then $10g(x) + 7 = x$, hence $g(x) = (x-7)/10$ **Ans**

(2)) Show that the function $f : R \to R$ given by $f(x) = x^3$ is injective. [Similar RBSE 2000]

Sol.one one/ many- one : $f(x) = x^3$

$f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$

hence f is one one , or **'f'** is injective . **Ans**

(3) Given a non empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows : For subset A,B in P(X) . ARB if and only if $A \subset B$. Is R an equivalence relation on P(X) ? justify your answer .

Sol. Given relation $ARB \Rightarrow A \subset B \,\forall\, AB \in P(X)$

Reflexive: $ARA \Rightarrow A \subset A$, each set is the subset of itself , hence **R** is reflexive relation .

Symmetric: $ARB \Rightarrow A \subset B$..........(1)

and $BRA \Rightarrow B \subset A$..........(2)

Both these relation can't be satisfying till A = B

Hence given relation is not symmetric.

Transitive : $ARB \Rightarrow A \subset B$..........(1) and

$BRC \Rightarrow B \subset C$..........(2)

$A \subset B \subset C \Rightarrow A \subset C \Rightarrow ARC$

Hence given relation is transitive.

Since R is reflexive, Skew symmetric and transitive then it is partial order relation. **Ans**

(4) Find the number of all onto function from the set {1, 2, 3, ..., n} to itself.

Sol. If function is defined as $f: X \to Y$, $X = Y = \{1, 2, 3 \ .... \ n\}$

one of the element of set Y (say 1) may have any one of the pre-images 1 , 2 ,3 ....n , i.e., in n ways .

the second element (say 2) will have the pre-images in (n-1) ways .

$\because$ the number of ways we can have the pre-images $n \times (n-1)(n-2).....3.2.1 = n!$

hence total number of one one onto mapping defined from X to Y is $n!$.

(5) Let A={-1, 0, 1, 2}, B = {-4, -2, 0, 2} and $f, g: A \to B$ be functions defined by $f(x) = x^2 - x,\ x \in A$

and $g(x) = 2\left|x - \frac{1}{2}\right| - 1,\ x \in A$.

Are f and g equal ? Justify your answer.

Sol. If $f, g: A \to B, f(x) = x^2 - x$, $g(x) = 2\left|x - \frac{1}{2}\right| - 1$

where $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$

two function will be equal function if domain , co-domain and range of function are equal

at $x = -1,\ f(-1) = (-1)^2 - (-1) = 2,$

$$g(-1) = 2\left|-1 - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 2$$

at $x = 0,\ f(0) = (0)^2 - 0 = 0$

$$g(0) = 2\left|0 - \frac{1}{2}\right| - 1 = 1 - 1 = 0$$

at $x = 1, f(1) = (1)^2 - 1 = 0,\ g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

at $x = 2, f(2) = (2)^2 - 2 = 2,\ g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 2$

Range of $f = \{0, 2\} =$ Range of g

$f(x) = g(x)\ \forall\ x \in A$,

hence the range of both functions are equal.

so both given functions are equal function. **Ans**

(6) Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(A) 1 (B) 2 (C) 3 (D) 4

[CBSE 2011 (Delhi)]

Sol. $A = \{1, 2\}$, $B = \{1, 3\}$

$R = \{(1,1)\ (1,3)\ (2,1)\ (1,2)\ (3,1)\ (2,2)\}$

Reflexive and symmetric not transitive . **Ans**

(7) Let A={1,2,3} , then number of equivalence relation containing (1,2) is

(a) 1 (b) 2 (c) 3 (d) 4

Sol. Given that A={1,2,3} if relation is equivalence then it must be reflexive , symmetric and transitive .

The smallest equivalence relation containing (1,2) is

$R_1 = \{(1,1)(2,2)(3,3)(1,2)(2,1)\}$

Now , if we add one pair (2,3) then we must have to add (3,2).

Since relation is symmetric , hence another relation is

$R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,1)(2,3)(3,2)\}$

Now , if we add one pair (1,3) then we must have to add (3,1).

Since relation is symmetric , hence another relation is

$R_3 = \{(1,1)(2,2)(3,3)(1,2)(2,1)(1,3)(3,1)\}$

Hence this show that the total number of equivalence relation containing {1,2}.

Thus alternative (B) is correct answer . **Ans**