Exercise 2.1 [NCERT]

Find the principle values of the following:

(1) $\sin^{-1}(-1/2)$ **[CBSE 2001 (D.B)]**

Sol. $\sin^{-1}(-1/2) = -\sin^{-1}(1/2) = -(+\pi/6)$ **Ans**

(2) $\cos^{-1}(\sqrt{3}/2)$ **[CBSE 2002]**

Sol. $\cos^{-1}(\sqrt{3}/2) = \theta \Rightarrow \cos\theta = \sqrt{3}/2 \Rightarrow \theta = \pi/6$ **Ans**

(3) $\operatorname{cosec}^{-1}(2)$

Sol. $\operatorname{cosec}^{-1}(2) = \theta \Rightarrow \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta = 1/2 \Rightarrow \theta = 30^0 = \pi/6$ **Ans**

(4) $\tan^{-1}(-\sqrt{3})$ **[CBSE 2003 (D.B)]**

Sol. $\tan^{-1}(-\sqrt{3}) = -\tan^{-1}\sqrt{3} = -\pi/3$ **Ans**

(5) $\cos^{-1}(-1/2)$ **[CBSE 2010 (AI)]**

Sol. $\cos^{-1}(-1/2) = \pi - \cos^{-1}(1/2) = \pi - \pi/3 = 2\pi/3$ **Ans**

(6) $\tan^{-1}(-1)$ **[CBSE 2011 (F), 2008 C, 2004(D.B)]**

Sol. $\tan^{-1}(-1) = -\tan^{-1}(1) = -\pi/4$ **Ans**

(7) $\sec^{-1}(2/\sqrt{3})$ **[CBSE 2002]**

Sol. $\sec^{-1}(2/\sqrt{3}) = \cos^{-1}(\sqrt{3}/2) = \pi/6$ **Ans**

(8) $\cot^{-1}(\sqrt{3})$

Sol. $\cot^{-1}(\sqrt{3}) = \tan^{-1}(1/\sqrt{3}) = \pi/6$ **Ans**

(9) $\cos^{-1}(-1/\sqrt{2})$

Sol. $\cos^{-1}(-1/\sqrt{2}) = \pi - \cos^{-1}(1/\sqrt{2}) = \pi - (\pi/4) = 3\pi/4$ **Ans**

Misconception : $\cos^{-1}(-1/\sqrt{2}) = \cos^{-1}(-\pi/4)$

or $\cos^{-1}(-1/\sqrt{2}) = \cos^{-1}(-\cos\pi/4) = -\pi/4$

Which is incorrect .

(10) $\operatorname{cosec}^{-1}(-\sqrt{2})$

Sol.

$$\operatorname{cosec}^{-1}(-\sqrt{2}) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$ **Ans**

(11) $\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$

[CBSE 2007(AI)]

Sol.

$$\tan^{-1}(1) + \pi - \cos^{-1}\frac{1}{2} - \sin^{-1}\frac{1}{2} = \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \left(\frac{\pi}{6}\right) = \frac{3\pi}{4}$$ **Ans**

(12) $\cos^{-1}(1/2) + 2\sin^{-1}(1/2)$ **[CBSE 2007(AI)]**

Sol. $\cos^{-1}(1/2) = \pi/3$, $\sin^{-1}(1/2) = \pi/6$

$$\cos^{-1}(1/2) + 2\sin^{-1}(1/2) = \frac{\pi}{3} + 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$ **Ans**

(13) If $\sin^{-1}x = y$ then

(a) $0 \le y \le \pi$ (b) $-\pi/2 \le y \le \pi/2$

(c) $0 < y < \pi$ (d) $-\pi/2 < y < \pi/2$

Sol. $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2} \Rightarrow -\pi/2 \le y \le \pi/2$ **Ans**(b)

(14) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) =$ **[CBSE 2012]**

(a) π (b) $-\pi/3$ (c) $\pi/3$ (d) $2\pi/3$

Sol. $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - \cos^{-1}(-1/2)$

$\tan^{-1}(\sqrt{3}) - [\pi - \cos^{-1}(1/2)]$

$= \pi/3 - [\pi - \pi/3] = \pi/3 - [2\pi/3] = -\pi/3$ **Ans** (b)

Exercise 2.2 [NCERT]

Prove the following:

(1) $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, $x \in [-1/2, 1/2]$

[Jammu 2013; H.P.B 2012, 10]

Sol. Let $\sin^{-1}x = \theta$, $x = \sin\theta$

we know that $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$\sin 3\theta = (3x - 4x^3) \Rightarrow 3\theta = \sin^{-1}(3x - 4x^3)$

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ **Ans**

(2) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, $x \in [1/2, 1]$

[Karnataka (B) 2014; H.P.B 2012, 10; CBSE 2004]

Sol. $\cos^{-1}x = \theta \Rightarrow x = \cos\theta$, we know that

$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta = 4x^3 - 3x$

$3\theta = \cos^{-1}(4x^3 - 3x)$, hence $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

Ans

Write the following functions in the simplest form :

(3) $\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$, $x\neq 0$

[CBSE 2000 (AI); Jammu 2014, 13; H.P.B 2013(S); P.B 2010]

Sol. $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$, put

$x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$$\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)=\tan^{-1}\left(\frac{2\sin^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}\right)$$

$\because\ 1-\cos\theta=2\sin^2(\theta/2)$,

$\sin\theta=2\sin(\theta/2)\cos(\theta/2)$

$=\tan^{-1}\tan\dfrac{\theta}{2}=\dfrac{\theta}{2}=\dfrac{1}{2}\tan^{-1}(x)$ **Ans**

(4) $\tan^{-1}\left(\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right)$, $x<\pi$ **[CBSE 2005 (Delhi)]**

Sol. $\tan^{-1}\left[\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right]=\tan^{-1}\left[\sqrt{\dfrac{2\sin^2(x/2)}{2\cos^2(x/2)}}\right]$

$=\tan^{-1}\sqrt{\tan^2(x/2)}=\tan^{-1}\tan(x/2)=x/2$

Ans

(5) $\tan^{-1}\left(\dfrac{\cos x-\sin x}{\cos x+\sin x}\right), 0<x<\pi$

[RBSE 2017 -2014 ; CBSE 2005 (AI); H.B 2012, 11; Jammu 2013; H.P.B 2012]

Sol.

$$\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right]=\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)=\tan^{-1}\tan\left(\frac{\pi}{4}-x\right)=\frac{\pi}{4}-x$$

Ans

(6) $\tan^{-1}\dfrac{x}{\sqrt{a^2-x^2}}$, $|x|<a$ **[CBSE 2005 (D.B) ; H.P.B 2013(S), 13, 10(S), 10]**

Sol. Put $x=a\sin\theta \Rightarrow \sin\theta=\dfrac{x}{a}$ or $\theta=\sin^{-1}(x/a)$

$$\tan^{-1}\left[\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right]=\tan^{-1}\left[\frac{a\sin\theta}{a\cos\theta}\right]=\tan^{-1}\tan\theta=\theta$$

hence $\tan^{-1}\dfrac{x}{\sqrt{a^2-x^2}}=\theta=\sin^{-1}\dfrac{x}{a}$ **Ans**

(7) $\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right)$, $a>0$; $\dfrac{-a}{\sqrt{3}}\le x\le\dfrac{a}{\sqrt{3}}$

[CBSE 2005 (AI); H.P.B 2013]

Sol. $\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right)$, Put $x=a\tan\theta \Rightarrow \tan\theta=x/a$

$$\tan^{-1}\left[\frac{3a^3\tan\theta-a^3\tan^3\theta}{a^3-3a^3\tan^2\theta}\right]=\tan^{-1}\left[\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right]$$

$\tan^{-1}\tan 3\theta=3\theta=3\tan^{-1}(x/a)$ **Ans**

Find the value of each of the following:

(8) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\dfrac{1}{2}\right)\right]$

Sol. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\dfrac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2\dfrac{\pi}{6}\right)\right]$

$\tan^{-1}\left[2\cos\left(\dfrac{\pi}{3}\right)\right]=\tan^{-1}\left[2\times\dfrac{1}{2}\right]=\tan^{-1}(1)=\dfrac{\pi}{4}$ **Ans**

(9) $\tan\dfrac{1}{2}\left[\sin^{-1}\dfrac{2x}{1+x^2}+\cos^{-1}\dfrac{1-y^2}{1+y^2}\right]$, $|x|<1, y>0$ and $xy<1$

[CBSE 2001 (AI)]

Sol. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)=2\tan^{-1}x$, $\cos^{-1}\left(\dfrac{1-y^2}{1+y^2}\right)=2\tan^{-1}y$

$\tan\dfrac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]=\tan\left[\tan^{-1}x+\tan^{-1}y\right]$

$=\tan\left[\tan^{-1}\left(\dfrac{x+y}{1-xy}\right)\right]=\dfrac{x+y}{1-xy}$

Ans

Find the values of each of the expressions in exercises 10 to15

(10) $\sin^{-1}(\sin(2\pi/3))$ **[IIT 1986; WBJEE 1995]**

Sol.

$\sin^{-1}\sin(2\pi/3)=\sin^{-1}\sin(\pi-(\pi/3))=\sin^{-1}\sin(\pi/3)=\pi/3$

Ans

(11) $\tan^{-1}[\tan(3\pi/4)]$ **[CBSE 2011(Delhi) , 2009 (F)]**

Sol. $\tan^{-1}[\tan(3\pi/4)]=\tan^{-1}\tan(\pi-(\pi/4))$

$=\tan^{-1}[-\tan(\pi/4)]\Rightarrow -\tan^{-1}\tan(\pi/4)=-\pi/4$

Ans

(12) $\tan\left(\sin^{-1}\dfrac{3}{5}+\cot^{-1}\dfrac{3}{2}\right)$

[RBSE 2012; IIT 1983; AIEEE 2008; DCE 2000]

Sol. $\because\ \sin^{-1}(3/5)=\tan^{-1}(3/4)$, $\cot^{-1}(3/2)=\tan^{-1}(2/3)$

$$\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)=\tan\left(\tan^{-1}\left(\frac{(3/4)+(2/3)}{1-(3/4)\cdot(2/3)}\right)\right)=\frac{9+8}{12}\times\frac{12}{6}=\frac{17}{6}$$

Ans

(13) $\cos^{-1}\left(\cos(7\pi/6)\right)=$

[CBSE 2009(AI); 2011 (DB); H.P.B 2013, 2011, 2010, 09]

(a) $7\pi/6$ (b) $5\pi/6$ (c) $\pi/3$ (d) $\pi/6$

Sol. $\cos^{-1}\cos\left(\frac{7\pi}{6}\right)=\cos^{-1}\cos\left(2\pi-\frac{5\pi}{6}\right)=\cos^{-1}\cos\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}$

Ans (b)

(14) $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)=$

[CBSE 2011 , 2008(DB); H.P.B 2013, H.B. 13, 2011, 10(S), 10]

(a) $1/2$ (b) $1/3$ (c) $1/4$ (d) 1

Sol $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]=\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]=\sin\left(\frac{\pi}{2}\right)=1$

Ans (d)

(15) $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)=$ **[CBSE 2013]**

(a) π (b) $-\pi/2$ (c) 0 (d) $2/\sqrt{3}$

Sol. $\tan^{-1}\left(\sqrt{3}\right)-\cot^{-1}\left(-\sqrt{3}\right)=\tan^{-1}\sqrt{3}-\left[\pi-\cot^{-1}\left(\sqrt{3}\right)\right]$

$$=\frac{\pi}{3}-\left(\pi-\frac{\pi}{6}\right)=\frac{\pi}{3}+\frac{\pi}{6}-\pi=-\frac{\pi}{2}$$ **Ans**(b)

Miscellaneous Exercise

Find the value of the following

(1) $\cos^{-1}\left(\cos(13\pi/6)\right)$

Sol $\cos^{-1}\cos(13\pi/6)=\cos^{-1}\cos\left(2\pi+\pi/6\right)$

$=\cos^{-1}\cos(\pi/6)=\pi/6$

(2) $\tan^{-1}\tan\left(7\pi/6\right)$

Sol. $\tan^{-1}\tan\left(\frac{7\pi}{6}\right)=\tan^{-1}\tan\left(\pi+\frac{\pi}{6}\right)$

$=\tan^{-1}\tan(\pi/6)=\pi/6$ **Ans**

(3) prove that $2\sin^{-1}\left(3/5\right)=\tan^{-1}(24/7)$ **[H.B 2012]**

Sol. We know that $2\sin^{-1}x=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$2\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(2\times\frac{3}{5}\sqrt{1-\frac{9}{25}}\right)=\sin^{-1}\left(\frac{6}{5}\sqrt{\frac{16}{25}}\right)$$

$$=\sin^{-1}\left(\frac{6}{5}\cdot\frac{4}{5}\right)=\sin^{-1}\left(\frac{24}{25}\right)=\tan^{-1}\left(\frac{24}{7}\right)$$ **Ans**

(4) $\sin^{-1}(8/17)+\sin^{-1}(3/5)=\tan^{-1}(77/36)$

[RBSE 2017]

Sol. Let $\sin^{-1}(8/17)=A$ and $\sin^{-1}(3/5)=B$ ....(1)

$\sin A=8/17$, $\sin B=3/5$

we know that $\sin^2\theta+\cos^2\theta=1$

$\cos^2\theta=1-\sin^2\theta\Rightarrow\cos\theta=\sqrt{1-\sin^2\theta}$

$$\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\left(\frac{8}{17}\right)^2}=\sqrt{\frac{289-64}{289}}=\frac{15}{17}$$

$$\cos B=\sqrt{1-\sin^2 B}=\sqrt{1-\left(\frac{3}{5}\right)^2}=\sqrt{\frac{25-9}{25}}=\frac{4}{5}$$

$$\tan A=\frac{\sin A}{\cos A}=\frac{8/17}{15/17}=\frac{8}{15}\Rightarrow A=\tan^{-1}\frac{8}{15}$$

$$\tan B=\frac{\sin B}{\cos B}=\frac{3/5}{4/5}=\frac{3}{4}\Rightarrow B=\tan^{-1}\frac{3}{4}$$

hence $\sin^{-1}\left(\frac{8}{17}\right)=\tan^{-1}\frac{8}{15}$, $\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\frac{3}{4}$

Now taking L.H.S $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4}$

$$=\tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}\right)=\tan^{-1}\left(\frac{32+45}{60-24}\right)=\tan^{-1}\frac{77}{36}$$ **Ans**

(5) $\cos^{-1}\left(4/5\right)+\cos^{-1}\left(12/13\right)=\cos^{-1}\left(33/65\right)$

[CBSE 2012, 2009 (C) (Delhi) ; 2010 (C), NDA 2012 PB 2010]

Sol. $\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left[xy-\sqrt{1-x^2}\sqrt{1-y^2}\right]$

$$\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\left[\frac{4}{5}\times\frac{12}{13}-\sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right]$$

$$=\cos^{-1}\left[\frac{48}{65}-\frac{3}{5}\times\frac{5}{13}\right]=\cos^{-1}\left(\frac{33}{65}\right)$$ **Ans**

(6) $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

[CBSE 2012 ,2010 (AI); PB 2010]

Sol L.H.S $\cos^{-1}(12/13)=A$, $\sin^{-1}(3/5)=B$, hence

$\cos A=12/13$, $\sin B=3/5$

$\sin A=\sqrt{1-\cos^2 A}=\sqrt{1-(12/13)^2}=5/13$

$\cos B=\sqrt{1-\sin^2 B}=\sqrt{1-(3/5)^2}=4/5$

Using formula :

$\sin(A+B)=\sin A\cos B+\cos A\sin B$

$$\sin(A+B)=\frac{5}{13}\times\frac{4}{5}+\frac{12}{13}\times\frac{3}{5}=\frac{20}{65}+\frac{36}{65}=\frac{56}{65}$$

$A+B=\sin^{-1}(56/65)$

Hence $\cos^{-1}(12/13)+\sin^{-1}(3/5)=\sin^{-1}(56/65)$ **Ans**

(7) $\tan^{-1}(63/16)=\sin^{-1}(5/13)+\cos^{-1}(3/5)$

[RBSE 2014 ; Similar CBSE 2009; H.P.B 2011]

Sol. RHS

$\sin^{-1}(5/13)=\tan^{-1}(5/12),\ \cos^{-1}(3/5)=\tan^{-1}(4/3)$

$\sin^{-1}(5/13)+\cos^{-1}(3/5)=\tan^{-1}(5/12)+\tan^{-1}(4/3)$

$\tan^{-1}\left(\frac{(5/12)+(4/3)}{1-(5/12)\cdot(4/3)}\right)=\tan^{-1}\left(\frac{15+48}{36-20}\right)=\tan^{-1}\left(\frac{63}{16}\right)$ **Ans**

(8) $\tan^{-1}(\sqrt{x})=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in[0,1]$

[CBSE 2010 (Delhi)]

Sol. we know that, $2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, Put $\sqrt{x}$, at the place of x

$2\tan^{-1}(\sqrt{x})=\cos^{-1}\left(\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right)$, hence

$\tan^{-1}(\sqrt{x})=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ **H.P**

(9) $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

[CBSE 2014, 2011 (Delhi) , 2009(AI) , 2009 (AI) , 2008 (DB) , 2006 C (AI) ; RPET 2005]

Sol $\sqrt{1\pm\sin x}=\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\pm 2\cos\frac{x}{2}\sin\frac{x}{2}}$

$=\cos\frac{x}{2}\pm\sin\frac{x}{2}$

$\cot^{-1}\left[\frac{(\cos(x/2)+\sin(x/2))+(\cos(x/2)-\sin(x/2))}{(\cos(x/2)+\sin(x/2))-(\cos(x/2)-\sin(x/2))}\right]$

$=\cos^{-1}\left[\frac{2\cos(x/2)}{2\sin(x/2)}\right]=\cot^{-1}\left[\cot\frac{x}{2}\right]=\frac{x}{2}$ **Ans**

(10) $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, -(1/\sqrt{2})\le x\le 1$

[RBSE 2015; CBSE 2014 (AI) , 2011 (AI) , 2006(Delhi)]

Sol. $\tan^{-1}\left(\frac{1-\sqrt{\frac{1-x}{1+x}}}{1+\sqrt{\frac{1-x}{1+x}}}\right)=\frac{\pi}{4}-\tan^{-1}\sqrt{\frac{1-x}{1+x}}$, put $x=\cos\theta$

$\frac{\pi}{4}-\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\frac{\pi}{4}-\tan^{-1}\sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}}$

$=\frac{\pi}{4}-\tan^{-1}\tan\left(\frac{\theta}{2}\right)=\frac{\pi}{4}-\frac{\theta}{2}=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$ **Ans**

(11) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

[CBSE 2009 (AI) , 2006 (D.B);AMU 2008]

Sol. $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

$\tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right)=\tan^{-1}\left(\frac{2}{\sin x}\right)\Rightarrow\frac{2\cos x}{\sin^2 x}=\frac{2}{\sin x}$

$2\sin x\cos x=2\sin^2 x\Rightarrow 2\sin x[\cos x-\sin x]=0$

$\Rightarrow \sin x=0\Rightarrow x=0\ or\ \pi$

$\cos x=\sin x\Rightarrow\cot x=1\Rightarrow x=\pi/4$ **Ans**

(12) $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x,(x>0)$

[RBSE 2017(Supp.);CBSE 2011(F), 2009(AI), 2009C(Delhi), CBSE 2008(AI) , 2008C(Delhi); H.B 2012]

Sol. We know that, $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$

$\frac{\pi}{4}-\tan^{-1}x=\frac{1}{2}\tan^{-1}x\Rightarrow\frac{\pi}{4}=\frac{3}{2}\tan^{-1}x$

$\tan^{-1}x=\pi/6\Rightarrow x=\tan(\pi/6)=1/\sqrt{3}$ **Ans**

(13) $\sin(\tan^{-1}x),|x|<1=$ **[VIT 2008]**

(a) $x/\sqrt{1-x^2}$ (b) $1/\sqrt{1-x^2}$

(c) $1/\sqrt{1+x^2}$ (d) $x/\sqrt{1+x^2}$

Sol. Let $\tan^{-1}x=\sin^{-1}\left(x/\sqrt{x^2+1}\right)$

$\sin\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{x}{\sqrt{x^2+1}}$ **Ans [D]**

(14) If $\sin^{-1}(1-x)-2\sin^{-1}x=\pi/2$ then $x=$

[BITSAT 2005; GCET 2007]

(a) $0,(1/2)$ (b) $1,(1/2)$ (c) 0 (d) $1/2$

Sol $\sin^{-1}(1-x)=\frac{\pi}{2}+2\sin^{-1}x\Rightarrow(1-x)=\sin\left[\frac{\pi}{2}+2\sin^{-1}x\right]$

$1-x=\cos(2\sin^{-1}x)\Rightarrow 1-x=\cos\left[2\left(\frac{\pi}{2}-\cos^{-1}x\right)\right]$

$1-x=-\cos(2\cos^{-1}x)\Rightarrow x-1=\cos\cos^{-1}(2x^2-1)$

$2x^2-x=0\Rightarrow x(2x-1)=0\Rightarrow x=0,\ 1/2$ **Ans**