# Kinematics

**Vectors:** $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

- Coplaner vectors:

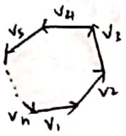
$\vec{a} = \lambda\vec{b} + \mu\vec{c}$.

- Addition of vectors:

$\vec{P} + \vec{R} = \sqrt{P^2 + R^2 + 2PR\cos\theta}$

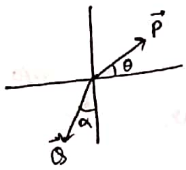
- Angle with P: $\tan\alpha = \frac{R\sin\theta}{P + R\cos\theta}$

- Polygon law:



$\vec{V_1} + \vec{V_2} + \dots + \vec{V_n} = 0$

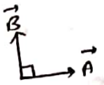
- Resolution of vectors:



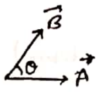
$\vec{P} = P\cos\theta\,\hat{i} + P\sin\theta\,\hat{j}$

$\vec{Q} = Q\cos\theta\,(-\hat{j}) + Q\sin\theta(-\hat{i})$

- Uses of dot product:
  - To prove perpendicularity $\vec{A}\cdot\vec{B} = 0$



  - To find angle b/w 2 vectors



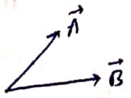
$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

- Uses of cross product:
  - To prove 2 vectors parallel. i.e. $\vec{A}\times\vec{B} = 0$
  - To find area of triangle, llgm or llopiped.

- Scalar Triple product:

$\vec{A}\cdot(\vec{B}\times\vec{C}) = 0$ [condition for co-planarity]

- Parallel component:



$\vec{A_{\parallel}} = |\vec{A}|\cos\theta\ \hat{B}$

$\vec{A_{\parallel}} = \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\cdot\vec{B}$

- Perpendicular component.

$\vec{A_{\perp}} = \vec{A} - \vec{A_{\parallel}}$

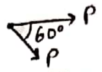
- Direction cosines: $l = \cos\alpha = \frac{x}{|\vec{r}|}$

$m = \cos\beta = \frac{y}{|\vec{r}|}$

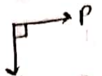
$n = \cos\gamma = \frac{z}{|\vec{r}|}$

$l^2 + m^2 + n^2 = 1$

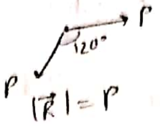
- Special cases:-



$|\vec{R}| = \sqrt{3}P$



$|\vec{R}| = \sqrt{2}P$



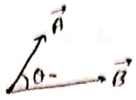
$|\vec{R}| = P$

- Subtraction (-ve addition)

$\vec{P} - \vec{Q} = \vec{P} + (-\vec{Q})$

- Dot product / Scalar product:

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$



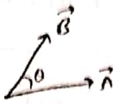
  - If, $\vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{B} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{A}\cdot\vec{B} = x_1x_2 + y_1y_2 + z_1z_2$

- Cross product / vector product

$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\ \hat{n}$



  - $\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{k}\times\hat{i} = \hat{j}$

  - If $\vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{B} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

↑ unit vector is perpendicular to both A & B.

# Kinematics

- Frame of reference (FOR):
  - Inertial FOR $(a=0)$
  - Non Inertial FOR $(a\neq0)$

- $v_{avg} = \frac{\Delta s}{\Delta t} = \frac{\int v\,dt}{\int dt}$

- Speed $= |\vec{v}|$
- $|$insta $v|$ = insta speed
- $v_{avg} \leq$ avg speed

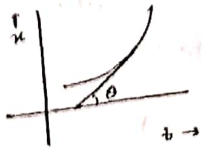
- Equation of Kinematics :
  - $v = u + at$
  - $s = s_0 + ut + \frac{1}{2}at^2$
  - $v^2 = u^2 + 2as$
  - $S_{nth} = u + \frac{1}{2}a(2n-1)$

The follower:

$t = \frac{lv}{v^2-u^2}$

- Distance : Actual length of the path
- Displacement : Shortest dist. b/w initial & final point

$\tan\theta = \langle v \rangle$



$\tan\theta = \frac{dx}{dt} = v_{insta}$

- Vertical motion

$T = \frac{2u}{g}$

$T = \sqrt{2gh}$

$H = \frac{u^2}{2g}$

- Relative velocity:

$v_{rel} = v_2 - v_1 = v_{2/1}$

2 w.r.t 1.

- Shortest Dist. Problem.

- converging polygon:

Use relative, side.

$t = \frac{a}{v(1-\cos 2\pi/n)}$

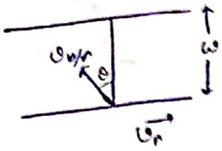
- Projectile on inclined plane:

change to $g\cos\theta$, $g\sin\theta$,

resolve everything into along and ⊥ to plane

work accordingly.

★ River boat Problem: $\vec{v}_{b/g} = \vec{v}_{b/r} + \vec{v}_r$



- Time $= \frac{\omega}{v_{b/r}\cos\theta}$ w/d

- Drift $= \frac{(v_r - v_{b/r}\sin\theta)\,\omega}{v_{b/r}\cos\theta}$ w/o

- shortest time $= \frac{\omega}{v_{b/r}}$ $[\cos\theta = 1, \theta = 0°]$

- No drift (only when $v_r \leq v_{b/r}$) : $\sin\theta = \frac{v_r}{v_{b/r}}$ → No matter what $\left[\sin\theta = \frac{\text{smaller}}{\text{larger}}\right]$

- When $v_r > v_{b/r}$, minimum drift → $\sin\theta = \frac{v_{b/r}}{v_r}$ →

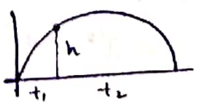
min drift, $x_{min} = \frac{\sqrt{v_r^2 - v_{b/r}^2}}{v_{b/r}}\,\omega$ w/d

- Projectile from height: Work out accordingly.

- Rain-Man: $\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$

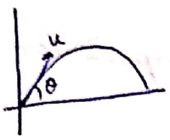
- To collide 2 projectiles: $u_1\sin\alpha = u_2\sin\beta$

- Particular height:



$h = \frac{gt_1t_2}{2}$

- Projectile: $T = \frac{2u\sin\theta}{g}$



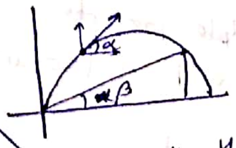
$R = \frac{u^2\sin 2\theta}{g} = \frac{2u_xu_y}{g}$

$H = \frac{u^2\sin^2\theta}{2g}$ $[R_\theta = R_{90-\theta}]$

- At any moment:

$\tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$

$\tan\beta = \frac{u\sin\theta t - \frac{1}{2}gt^2}{u\cos\theta t}$



basically $\frac{y\text{ comp}}{x\text{ comp}}$

- $\frac{H}{R} = \frac{\tan\theta}{4}$
- $H_{max} = R/4$ for $\theta = 45°$

- $\langle v \rangle$ for Projectile:
  - for $T$, $\langle v \rangle = u\cos\theta$
  - for $T/2$, $\langle v \rangle = \frac{u}{2}\sqrt{3\cos^2\theta + 1}$

- Equation of trajectory:

$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$ / $y = x\tan\theta(1 - x/R)$

- Max range for projectile on incline:

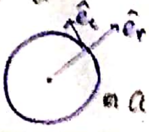
$\alpha = \pi/4 - \beta/2$

Circular motion:

- $s = r\theta$
- $\vec{v} = \vec{\omega} \times \vec{r}$
- $\vec{a} = \vec{\alpha} \times \vec{r}$

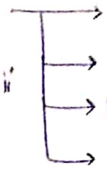
- $\langle \omega \rangle = \frac{\Delta\theta}{\Delta t}$
- $\omega = \frac{d\theta}{dt}$
- $\langle \omega \rangle = \frac{\int \omega\,dt}{\int dt}$
- $\alpha = \frac{d\omega}{dt}$

- Radial & Tangential accn.
  - $a_t = \frac{d|v|}{dt}$
  - $a_r = \frac{v^2}{r} = \frac{\omega^2}{r}$
  - $a_t = r\alpha$
  - $a = \sqrt{a_t^2 + a_r^2}$



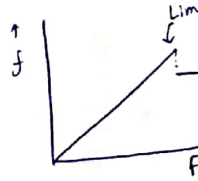
- $a = r\alpha\hat{e}_t - \omega^2 r\hat{e}_r$

- Force



iii) > i) > iv) > ii)

- Normal Reaction
- Tension: Direc
  - Rassi
- Newton's 1st La
- Newton's 3rd L
- Torque: $\vec{\tau} = \vec{r}$
- Newtons 2nd La
- Movable pulley:
- Spring Combos:
- Virtual Work Met
- Constraints: rela
- Wedge Constrain
- String Constraint
- Rod Constraints:
- Spring- String C
- Friction is an
- Allways acts
- In impending
- Frictional force



# NLM

⊘ Force → Electromagnetic (i)
→ Gravitational (ii)
→ Hadronic (Strong nuclear) (iii)
→ Weak nuclear force (iv)

iii) > i) > iv) > ii

⊘ Force → Contact force e.g. Friction, Normal
→ Feild force e.g. gravitation
→ Force through attachment e.g. Tension

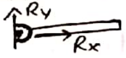
⊘ Normal Reaction: ▨ Acts because of hardness of surface ▨ Always along ⊥ to the surface

⊘ Tension: ▨ Direction of tension is always away from the body.
▨ Rassi ek toh tension ek! Jabtak rassi nehi badlegi, tension nehi badlega.

⊘ Newton's 1st Law: ▨ Equillibrium: $F_{net} = 0$ ▨ Rotational: $\tau_{net} = 0$

⊘ Newton's 3rd Law: ▨ Force always exist in pairs ▨ Action & Reaction occurs simultaneously.
▨ Action & Reaction acts on Different bodies.

⊘ Hinge reaction:



Consider two hinge reactions.

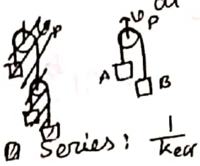
⊘ Torque: $\vec{\tau} = \vec{r} \times \vec{F} = rF_{\perp} = Fr_{\perp}$

⊘ Newtons 2nd Law: $F = \frac{d\vec{p}}{dt} = ma$

⊘ Simple pulley block:

$a = \frac{g(m_1 - m_2)}{\Sigma m}$

⊘ Movable pulley:

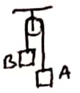


| $v_A + v_B = 2v_P$

⊘ Pseudo force: ▨ acts oppsite to accn.
▨ $F_P = ma_2$ → accn of non intial frame where the observer is in.

⊘ Spring Combos: ▨ Series: $\frac{1}{K_{eq}} = \Sigma \frac{1}{K_i}$ ▨ Parallel: $K_{eq} = \Sigma K_i$

⊘ Virtual Work Method:



VWM, $T\,dS_B = T\,dS_A$ [transform into $v$ or $a$]

⊘ Constraints: relative motion is forbidden in a certain directions.

⊘ Wedge Constraints:

Relative motion ⊥ to wedge is forbidden.

⊘ String Constraints: Relative velocity b/w two points along the length of string should be zero.

⊘ Rod Constraints: " " " " " " " " rod " " "

⊘ Spring- String Constraints: Spring needs some time to react.

## Friction

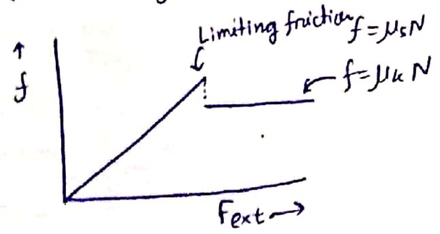
⊘ Friction is an Electromagnetic force, that is more electric and less magnetic in nature. ⊘ Self Adjusting Force.

⊘ Allways acts opposite to the direction of net velocity. ⊘ $f = \mu N$

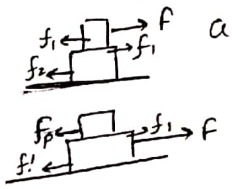
⊘ In impending state, the frictional force is ∝ the normal reaction. [$\mu \leq 1$]

⊘ Frictional force is independant of the area of surface in contact.

⊘



⊘ 2 body system:



$a = \frac{F}{m+M}$ if $f_1 \gg f_2$

To check, $a = \frac{F}{\Sigma m}$

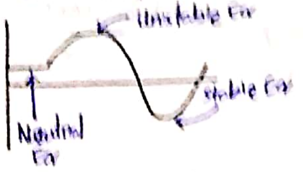
$F_P = ma$ if $F_P < F_{max}$ no slipping.

# WEP

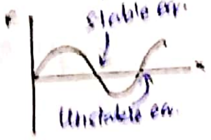
- Work: $W = \vec{F}\cdot\vec{s} = \int \vec{F}\cdot d\vec{s} = Fs\cos\theta$ [$\theta \to$ acute : ⊕ve W, $\theta \to$ obtuse : ⊖ve work]
  - W by Normal reaction = W by tension = 0
- The work done by any force is independant of the work done by other forces.
- The work done by a force is independant of the time in which the displacement has been brought about.
- The work done by force is always frame-dependant.
-  $W = Fl$
- Conservative force: Work done doesn't depend on path followed. e.g. Electrostatic, gravitational
- Non-conservative force: opposite. e.g. frictional force, magnetic force.
- Grad: $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ [$\vec{\nabla} \to$ Divergence of field, if $\vec{\nabla}\times\vec{F} = 0$, field in conservative]
- F-x Graph:

Area under F-x curve = Work

- Kinetic Energy: $KE = \frac{1}{2}mv^2 = p^2/2m$ [always ⊕ve, frame dependant]
- Momentum: $p = mv = m\frac{dx}{dt} = \sqrt{2mKE}$
- Potential Energy: → Elastic PE; → Gravitational PE → $U = mgh$
- PE is always collected w.r.t. a reference frame.
- WET: $W_{cons} + W_{non\text{-}cons} + W_{internal} + W_{external} + W_{pseudo} = \Delta KE$
- Work done by static friction is always 0.
-  $W_f = -\mu mgb$.
- Work done by spring force: $U = \frac{1}{2}kx^2$
- 
- Relation b/w force and PE: $F = -\frac{dU}{dx}$ / $\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$
- Power: $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$
- U-X Graph:



- F-x Graph.



- Radius of Curvature:

$$r = \frac{mv^2}{F_\perp}$$

$$\boxed{r = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{d^2y/dx^2}}$$

$y = f(x)$

General eq

- Conical Pendulum:

$\tan\theta = \frac{u^2}{gl\sin\theta}$

- Circumferencial tension:

$T = \frac{m\omega^2 R}{2\pi}$

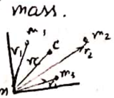
- Vertical Circle:
  - v at top to complete circle → $v = \sqrt{rg}$
  - To complete $\frac{1}{4}$th circle → $u = \sqrt{2gr}$
  - To complete circle → $u = \sqrt{5gr}$
  - If $\sqrt{2gr} < u < \sqrt{5gr}$

$\cos\theta = \frac{u^2 - 2gr}{3gr}$

- Rod to complete circle → $u = \sqrt{4gr}$

# COM

- COM : A point lying outside or inside a body where the whole mass of the body can be assumed to be concentrated. ~~The~~
- The concept of COM is based on the 1st moment of mass.
- COM of 2 body system ; Sum of moments about COM is zero
- Multiple body :- $r_c = \frac{\Sigma m_i r_i}{\Sigma m_i}$



- continuous mass distribution : linear → $\lambda = m/l$ ; superficial → $\sigma = \frac{m}{A}$ ; volumetric → $\rho = \frac{m}{V}$
- For continuous mass distribution: $x_c = \frac{\int x\,dm}{\int dm}$ , $y_c = \frac{\int y\,dm}{\int dm}$ , $z_c = \frac{\int z\,dm}{\int dm}$ | - $v_c = \frac{\Sigma m_i v_i}{\Sigma m_i}$ , $a_c = \frac{\Sigma m_i a_i}{\Sigma m_i}$

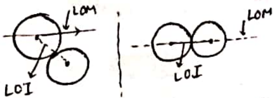
- Special objects:
  - → $l/2 = x_c$
  - Ring → $y_c = 2R/\pi$
  - Disc → $y_c = \frac{4R}{3\pi}$
  - Hollow hemis → $y_c = R/2$
  - Solid Hemis → $y_c = \frac{3R}{8}$
  - Solid cone → $y_c = 3/4\, h$ (from Top)
  - Hollow cone → $y_c = \frac{2h}{3}$ (from Top)

- Non uniform mass distribution: $\lambda = ax+b$ , $x_c = \frac{\int x(ax+b)\,dx}{\int (ax+b)\,dx}$
- Conservation of momentum: $\frac{d\vec{p}}{dt} = 0$ when $F_{net} = 0$
- All displacements, velocities and accns are to be taken w.r.t. ground.
- If net external force acting on a system is 0 in a certain direction, momentum is consered in that direction.
- COM CONSERVATION:

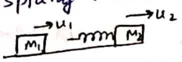
Take $dm/M = x$ , $dm/g = dm + x\,mdm + MdM = 0$

- Perfectly inelastic Collision $[e=0]$ $m_1u_1 + m_2u_2 = (m_1+m_2)v$
- momentum conservation in collision: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- $e = \frac{v_2 - v_1}{u_1 - u_2}$ ($v_2 - v_1$: $V_r$ of seperation; $u_1 - u_2$: $V_r$ of approach)

- Collisions:



- spring Model:



At max compression, $v_1 = v_2 = v$

- Elastic collision: - $v_2 - v_1 = u_1 - u_2$ $[e=1]$
  - $v_1 = \frac{m_1 - m_2}{m_1+m_2} u_1 + \frac{2m_2u_2}{\Sigma m}$
  - $v_2 = \frac{m_2 - m_1}{\Sigma m} u_2 + \frac{2m_1u_1}{\Sigma m}$

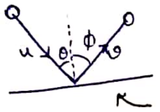
- Inelastic Collision: $[0<e<1)$
  - $v_1 = \frac{m_1 - em_2}{\Sigma m} + \frac{(1+e)m_2u_2}{\Sigma m}$
  - $v_2 = \frac{m_2 - em_1}{\Sigma m} + \frac{(1+e)m_1}{\Sigma m}u_1$

- f-t graph

- Special cases:
  - $m_1 = m_2 \longrightarrow v_1 = u_2$ / $v_2 = u_1$ [Exchange of v]
  - $m_1 >>> m_2$ , ~~$u_1$~~ $u_2 = 0 \longrightarrow v_2 = 2u_1$ $[\frac{m_2}{m_1} \approx 0]$
  - $m_1 <<< m_2$ , $u_2 = 0 \longrightarrow v_2 = 0 / v_1 = -u_1$ $[\frac{m_1}{m_2} \approx 0]$

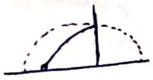
- $e = \frac{V_r \text{ of seperation along LOI}}{V_r \text{ of approach along LOI}}$

-



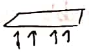
  - If friction $=0$, then $v$ comp ∥ to the surface remains unchanged, $u\sin\theta = v\sin\phi$
  - $\Delta P = 2mu\cos\theta$
  - $v\cos\phi = eu\cos\phi$
  - $\phi = \tan^{-1}\left(\frac{\tan\theta}{e}\right)$

- [No friction]



As ~~y~~ $v_y$ doesn't change.
$\therefore t_1 + t_2 = T$

- Oblique Collision: Components of velocities perpendicular to the Line of Impact remains unchange and along Line of impact, Formulae for head-on collision holds.

- Impulse: $\vec{J} = \Delta\vec{P} = m\vec{v_f} - m\vec{v_i} = m\Delta v$
  -  $\vec{J} = 2mu\cos\theta$
  -  $F = \Delta P\left(\frac{dN}{dt}\right)$, $F = mg$
- Impulsive force: When a large force acts for a small duration of time, then it is called impulsive. The force should nessecerily first increase to a large magnitude and decrease abruptly.
- Friction resulting from impulsive normal is also impulsive in nature.
- Weight (mg) and spring force are always non impulsive.
- Variable mass: $F_{th} = -v_r \frac{dm}{dt}$
- Rocket Propultion: $v = u + v_r \ln\left(\frac{m}{m_0}\right) - gt$
- $v = \frac{um_0}{m_0 + \lambda t}$

-

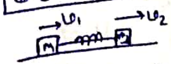
$F = v\frac{dm}{dt}$

$\boxed{F_{th} = \rho A v^2}$

- KE in ground frame:

$KE_G = KE_C + K$ of C frame
$= \frac{1}{2}\mu v_{rel}^2 + \frac{1}{2}(m_1+m_2)v_c^2$

- C-frame



$\vec{v_c} = \frac{m_1v_1 + m_2v_2}{m_1+m_2}$

$\vec{v_{1/c}} = \frac{m_2(v_1 - v_2)}{m_1+m_2}$

$\vec{v_{2/c}} = \frac{m_1(v_2 - v_1)}{m_1+m_2}$

$\boxed{\vec{P_{1/c}} + \vec{P_{2/c}} = 0}$

↑ C-frame also 0 momentum system.

- Kinetic Energy in C frame

$KE_C = \frac{1}{2}\mu v_{rel}^2$

Reduced mass $\left(\frac{mM}{m+M}\right)$

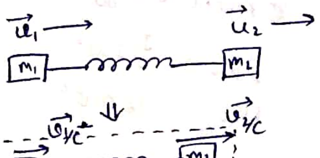
- Max compression:

$\frac{1}{2}\mu v_{rel}^2 = \frac{1}{2}kx_m^2$

- Max height:

$\frac{1}{2}\mu v^2 = mgh$

-



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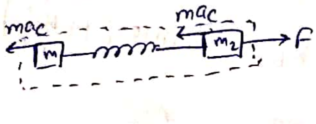
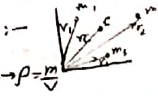


# COM

- COM : A point lying outside or inside a body where the whole mass of the body can be assumed to be concentrated. ~~The~~
- The concept of COM is based on the 1st moment of mass.
- COM of 2 body system : Sum of moments about COM is zero
- Multiple body :-



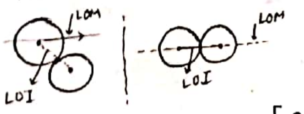
$r_c = \frac{\Sigma m_i r_i}{\Sigma m_i}$

- continuous mass distribution : linear → $\lambda = m/l$ ; superficial → $\sigma = \frac{m}{A}$ ; volumetric → $\rho = \frac{m}{V}$
- For continuous mass distribution : $x_c = \frac{\int x\,dm}{\int dm}$ , $y_c = \frac{\int y\,dm}{\int dm}$ , $z_c = \frac{\int z\,dm}{\int dm}$ | $v_c = \frac{\Sigma m_i v_i}{\Sigma m_i}$ , $a_c = \frac{\Sigma m_i a_i}{\Sigma m_i}$
- Special objects :
  - → $l/2 = x_c$
  - Ring → $y_c = 2R/\pi$
  - Disc → $y_c = \frac{4R}{3\pi}$
  - Hollow hemis → $y_c = R/2$
  - Solid Hemis → $y_c = \frac{3R}{8}$
  - Solid cone → $y_c = 3/4\,h$ (from Top)
  - Hollow cone → $y_c = \frac{2h}{3}$ (from Top)
- Non uniform mass distribution : $\lambda = ax+b$ , $x_c = \frac{\int x(ax+b)\,dx}{\int (ax+b)\,dx}$
- Conservation of momentum : $\frac{d\vec{p}}{dt} = 0$ when $F_{net} = 0$
- All displacements, velocities and accns are to be taken w.r.t. ground.
- If net external force acting on a system is 0 in a certain direction momentum is consered in that direction.
- COM CONSERVATION :

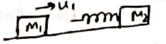
Take $d_{m/M} = x$ , $d_{m/g} = d_m + x$ $md_m + Md_M = 0$

- Perfectly inelastic Collision $[e=0]$ $m_1u_1 + m_2u_2 = (m_1+m_2)v$
- $e = \frac{v_2 - v_1}{u_1 - u_2}$ ($v_r$ of seperation over $v_r$ of approa)

## Collisions :

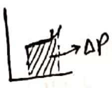


- Spring Model :



At max compression, $v_1 = v_2 = v$

- momentum conservation in collision : $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- Elastic collision : $v_2 - v_1 = u_1 - u_2$ $[e=1]$
  - $v_1 = \frac{m_1 - m_2}{m_1+m_2}u_1 + \frac{2m_2u_2}{\Sigma m}$
  - $v_2 = \frac{m_2 - m_1}{\Sigma m}u_2 + \frac{2m_1u_1}{\Sigma m}$
- Inelastic Collision : $[0<e<1)$
  - $v_1 = \frac{m_1 - em_2}{\Sigma m} + \frac{(1+e)m_2u_2}{\Sigma m}$
  - $v_2 = \frac{m_2 - em_1}{\Sigma m} + \frac{(1+e)m_1}{\Sigma m}u_1$
- f-t graph



- Special cases :
  - $m_1 = m_2 \longrightarrow v_1 = u_2$ / $v_2 = u_1$ [Exchange of v]
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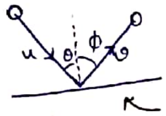
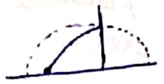
- $e = \frac{V_r \text{ of seperation along LOI}}{V_r \text{ of approach along LOI}}$



- If friction $=0$, then $v$ comp ∥ to the surface remains unchanged, $u\sin\theta = v\sin\phi$
- $\Delta P = 2mu\cos\theta$
- $v\cos\phi = eu\cos\phi$
- $\phi = \tan^{-1}\left(\frac{\tan\theta}{e}\right)$



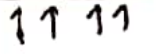
[No friction]
As ~~y~~ $v_y$ doesn't change.
$\therefore t_1 + t_2 = T$

- Oblique Collision : Components of velocities perpendicular to the Line of Impact remains unchan and along Line of impact , Formulae for head-on collision holds.

## Impuse :

$\vec{J} = \Delta\vec{P} = m\vec{v_f} - m\vec{v_i} = m\Delta v$

$\vec{J} = 2mu\cos\theta$



$F = \Delta P\left(\frac{dN}{dt}\right)$, F=mg

- When a large force acts for a small duration of time, then it is called impulsiv
- Impulsive force : When a large force should necessarily first increase to a large magnitude and decrease abruptly. The force should nessecerily first increase to a large magnitude and decrease abruptly.
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## Variable mass :

$F_{th} = -v_r\frac{dm}{dt}$

- Rocket Propultion : $v = u + v_r\ln\left(\frac{m}{m_0}\right) - gt$
- $v = \frac{um_0}{m_0 + \lambda t}$

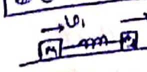
$F = v\frac{dm}{dt}$

$\boxed{F_{th} = \rho A v^2}$

- KE in ground frame :

$KE_G = KE_C + K$ of C frame
$= \frac{1}{2}\mu v_{rel}^2 + \frac{1}{2}(m_1+m_2)v_c^2$

## C-frame



$\vec{v_c} = \frac{m_1v_1 + m_2v_2}{m_1+m_2}$

$\vec{v}_{1/c} = \frac{m_2(v_1 - v_2)}{m_1+m_2}$

$\vec{v}_{2/c} = \frac{m_1(v_2 - v_1)}{m_1+m_2}$

$\boxed{\vec{P}_{1/c} + \vec{P}_{2/c} = 0}$

C-frame also 0 momentum system.

## Kinetic Energy in C Frame

$KE_C = \frac{1}{2}\mu v_{rel}^2$

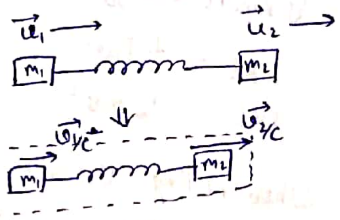
Reduced mass $\left(\frac{mM}{m+M}\right)$

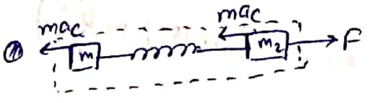
- Max compression :

$\frac{1}{2}\mu v_{rel}^2 = \frac{1}{2}kx_m^2$

- Max height :

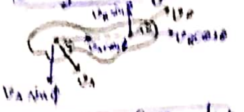
$\frac{1}{2}\mu v^2 = mgh$





# Rigid Body Dynamics

**Rigid Body** Relative velocity b/w any two points along the line joining them should be equal to 0.



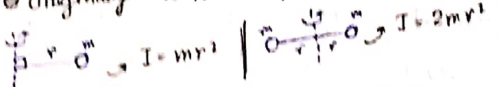
$|v_A \cos\phi - v_B \cos\theta = 0|$  $\omega = \frac{v_B \text{ in } \perp \text{ dir}}{AB} = \frac{v_B \sin\theta + v_A \sin\phi}{AB}$

**Moment of Inertia** It is the measure of rotational inertia. Depends on (i) Distribution of mass (ii) Choice of Axis of rotation (r)

$\boxed{I = \int r^2 . dm}$

- It is the 2nd moment of mass.
- Originally a tensor quantity but to be treated as a scalar.

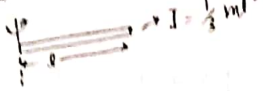
**Special Cases:**



$I = mr^2$ | $I = 2mr^2$

**Rod** →

- About end,



$I = \frac{1}{3} ml^2$

- About com

$I = \frac{1}{12} ml^2$

- About end com (slanted)

$I = \frac{1}{3} ml^2 \sin^2\theta$

**Ring** →

$I = mR^2$

**Disc** →

$I = \frac{1}{2} mR^2$

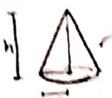
**Hollow sphere** →

$I = \frac{2}{3} mR^2$

**Solid sphere** →

$I = \frac{2}{5} mR^2$

**Solid Cone** →



$I = \frac{3}{10} mR^2$

**Hollow cone** → ring

$I = mR^2$

**Solid Cone** → Disc

$I = \frac{1}{2} mR^2$

**Hollow cone** →

$I = \frac{1}{2} mR^2$

**Segments** →

same as disc $I = \frac{1}{2} mR^2$

$I = \frac{2}{5} mR^2$

- Formula of MOI is same as that of original figure if the mass distribution is unchanged.

- $I_{ring} > I_{HS} > I_{Disc} > I_{SC} > I_{SC}$

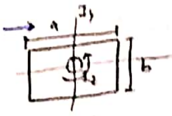
**Parallel Axis Theorem** The moment of inertia of a body about an axis parallel to the axis passing through COM, is equal to the sum of MOI about COM and the product of the mass and square of perpendicular dist. between two axes.

- It is applicable for all bodies.

**Perpendicular Axis Theorem** $I_z = I_x + I_y$

- Applicable only for plane lamina.

**Rectangular lamina** →



$I_x$ $I = \frac{m}{12}(a^2 + b^2)$

**Square lamina** →

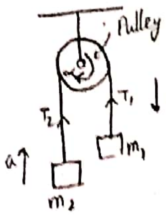
$I = \frac{1}{6} ma^2$

**Total Acceleration**

- $a = \underbrace{\vec{\omega} \times \vec{v}}_{radial} + \underbrace{\vec{\alpha} \times \vec{r}}_{tangential}$
- $a = \frac{d\vec{v}}{dt} = \frac{d(\vec{\omega} \times \vec{r})}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r}$

**Torque** $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$ • $\boxed{\tau = I\alpha}$

**Rotational constraint**



$m_1 g - T_1 = m_1 a$

$T_2 - m_2 g = m_2 a$

$T_1 R - T_2 R = I\alpha$

**Kinetic Energy**

RKE $= \frac{1}{2} I\omega^2$

TKE $= \frac{1}{2} mv^2$

When rolling, $v = \omega r$

radius of gyration.

$\therefore K_{tot} = \frac{1}{2} mv^2 \left(1 + \frac{k^2}{r^2}\right)$

$\therefore \frac{TKE}{K_{tot}} = \frac{r^2}{r^2 + k^2}$

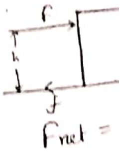
$\therefore \frac{RKE}{K_{tot}} = \frac{k^2}{r^2 + k^2}$

**Radius of Gyration** It is the effective dist from axis of rotation where the whole mass of the body can be assumed to be constructed so that it gives the same MOI as that of the original body.

$\boxed{K = \sqrt{\frac{I}{m}}}$ & $\boxed{I = mk^2}$

---

**Toppling:**



$F_{net} =$

When

$f = \mu N$ an

**Angular**

**Cases:**

$\vec{L}$

$\vec{L} = \vec{r} \times$

$\frac{d\vec{L}}{dt} = \vec{r} \times$

$= \vec{r}$

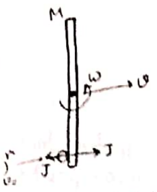
$= \vec{\tau}$

$\frac{d\vec{L}}{dt} = \vec{\tau}$

- If $\tau_{net} = 0$.

**Angular I**

$\vec{m} = \vec{r} \times$



Point immid

$v = \omega$

$x \to$ dis

be equal to 0.

in $\phi$

of mass
Axis of rotation (r)

(slanted)

$\frac{1}{3}ml^2\sin^2\theta$

$\rightarrow$

$I = \frac{2}{5}mR^2$

$\nearrow I = \frac{1}{2}mR^2$

Disc

original figure,

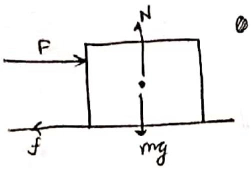
e axis passing
the product of
axes.

$I = \frac{1}{6}ma^2$

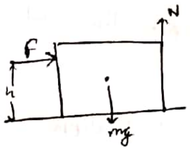
The effective
where the
ly can be
led so that
as that of

---

## Toppling:



- For sliding → $F > \mu mg$

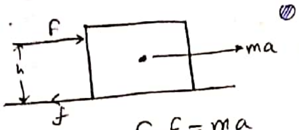


For toppling,

$Fh \geq mg\frac{b}{2}$

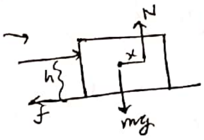
$F \geq \frac{mgb}{2h}$

Body will ~~slide~~ topple before sliding.



$F_{net} = F - f = ma$

- Sliding + toppling.
- Use torque balancing about COM in this case. →



$\boxed{F(h - \frac{b}{2}) + f\frac{b}{2} = Nx}$

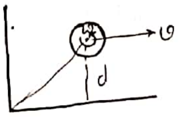
- Whenever toppling occurs,

$f = \mu N$ and $N = mg$ doesn't hold.

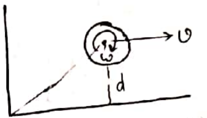
## Angular momentum

$\boxed{\vec{L} = \vec{r} \times \vec{p}} = rP_\perp = Pr_\perp$ || $\boxed{\vec{L} = \underset{\text{orbital}}{\vec{L_o}} + \underset{\text{spin}}{\vec{L_s}}}$

**Cases:**



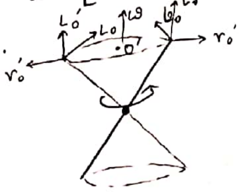
$L = mvd - I\omega$
[$L_o$ and $L_s$ are anti ||]



$L = mvd + I\omega$ [$L_o$ and $L_s$ are ||]

- $\vec{L} = I\vec{\omega}$ [vector form doesn't hold for asymetric rotation]

e.g.



$\vec{L} \neq I\vec{\omega}$ about O . but,

$\vec{L} = I\vec{\omega}$ about O.

- $\vec{L} = \vec{r} \times \vec{p}$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$= \vec{r} \times \vec{F} + 0$$

$$= \vec{\tau}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau},$$

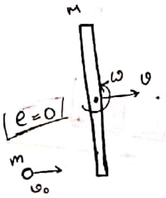
- If $\tau_{net} = 0$, $\vec{L} = $ const.

- $L = I\omega = $ const. [Iceskater]

$\frac{2\pi}{T} \propto \omega \propto \frac{1}{I}$

$\therefore \boxed{T \propto I}$

## Rotational Collision



Linear momentum conservation,

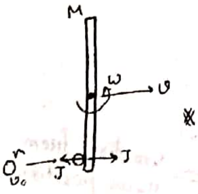
$mv_o = Mv + mv$

Angular momentum conservation

$mv_o\frac{l}{2} = \frac{Ml^2}{12}\omega + m(v + \frac{\omega l}{2})\frac{l}{2}$

## Angular Impulse

$\boxed{\vec{m} = \vec{r} \times \vec{J}}$



$\boxed{\vec{m} = \Delta\vec{L}}$

* $\frac{l}{2} \cdot J = \frac{Ml^2}{12}\omega$

Point immidiately at rest

$v = \omega x$

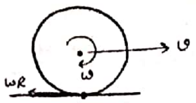
$x \rightarrow$ dist from com

Scanned by CamScanner

30

## Pure Rolling

For pure rolling to take place the relative velocity b/w two points at the point of contact should always be 0.



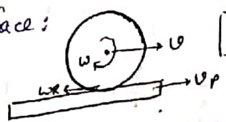
$\boxed{v = \omega R}$

$\boxed{f = \text{static} \neq \mu mg}$

$\boxed{W_f = 0}$

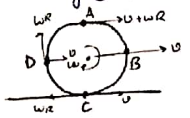
If, $v > \omega R$ → Fwd slipping or Bwd English
If, $v < \omega R$ → Bwd slipping or Fwd English
] → $f = \mu mg$

- On a movablog surface:



$\boxed{v - \omega R = v_P}$ $\boxed{W_f \neq 0}$

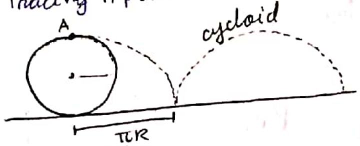
- Velocity of points:



$v_A = 2v$, $v_B = v\sqrt{2}$
$v_C = 0$, $v_D = v\sqrt{2}$

$\boxed{v_P = 2v \sin\frac{\theta}{2}}$

- Tracing A point.



In one revolution

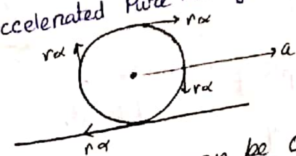
$v = 2v_0 \sin\frac{\omega t}{2}$

$$\int dx = 2v_0 \int_0^{2\pi/\omega} \sin\frac{\omega t}{2}\, dt$$

$$x = \frac{2v_0}{\omega/2}\left[\cos\frac{\omega t}{2}\right]_0^{2\pi/\omega}$$

$$\boxed{x = 4R \cdot 2 = 8R}$$

- Accelerated Pure Rolling

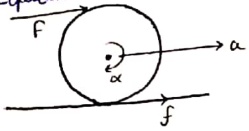


$a = r\alpha$

# Net accn can be 0 in—
→ 4th qud if accelerating
→ 1st qud if retarding

Ø The direction of friction is such so as to support pure rolling.

- Equations



$a = r\alpha$

$F + f = ma$ —①

$(F - f)r = I\alpha$

$F - f = \frac{Ia}{r^2}$

$$\frac{F+f}{F-f} = \frac{mr^2}{I}$$

$$\frac{F}{f} = \left(\frac{mr^2+I}{mr^2-I}\right)$$

$$f = \left(\frac{mr^2-I}{mr^2+I}\right)F$$

Ø If $I > mr^2$
$f \to -ve$
(Not possible)

If $I = mr^2$ (Ring)
$f = 0$

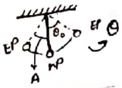
## SHM

### Linear SHM

→ $x = A\sin(\omega t + \phi)$

frequency
- $\omega = 2\pi f$
→ $\omega = \frac{2\pi}{T}$

### Angular SHM

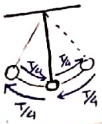


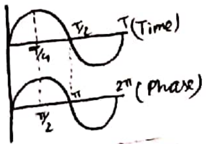
$\theta = \theta_0 \sin(\omega t + \phi)$

**Phase** We cannot put a 'time' as θ in sinθ, so we have to change ~~it~~ into terms of phase. [when starting from EP, $\theta = \theta_0 \sin(\omega t + \frac{\pi}{2})$]

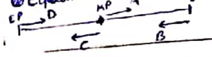
→ At EP, $\phi = \frac{\pi}{2}$

→ $x = A\sin(\omega t + \phi)$ ← Initial Phase or Epoch.





- Equations.



- A → $x = A\sin\omega t$
- B → $x = A\sin(\omega t + \frac{\pi}{2}) = A\cos\omega t$
- C → $x = -A\sin\omega t$
- D → $x = -A\cos\omega t$

### Velocity & Accn

→ $v = A\omega\cos\omega t$

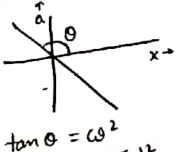
velocity Amp → $= v_0 \cos\omega t$

∴ $\omega = \frac{v_0}{A}$

→ $a = -A\omega^2 \sin\omega t$
$a = -\omega^2 x$

→ $v = A\omega\sqrt{1 - \sin^2\omega t}$
$= A\omega\sqrt{1 - \frac{x^2}{A^2}}$
$v = \omega\sqrt{A^2 - x^2}$

- Graph ($a = -\omega^2 x$)



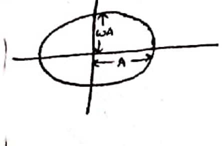
$\tan\theta = \omega^2$
$= \left(\frac{2\pi}{T}\right)^2$

$T = \frac{2\pi}{\sqrt{\tan\theta}}$

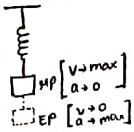
- Graph ($v = \omega\sqrt{A^2 - x^2}$)

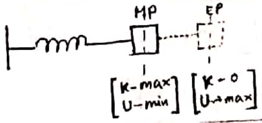
$\frac{v^2}{\omega^2} = A^2 - x^2$

$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$



### Energetics

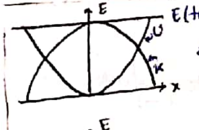




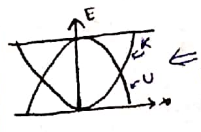
→ $KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$
$\langle KE \rangle = \frac{1}{4} m\omega^2 A^2$

→ $U = \frac{1}{2} m\omega^2 A^2 \sin^2\omega t$
$\langle U \rangle = \frac{1}{4} m\omega^2 A^2$

→ $TE = \frac{1}{2} m\omega^2 A^2$ [Conserved]

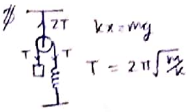


⇐ Starting from mean positio

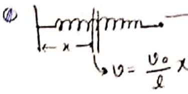


⇐ Starting from Extreme positio

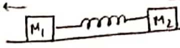
- Method to find
- Draw the FBD
- Displace in slight
- Find ~~net~~ net restoring
- apply $a = \omega^2 x$ or



$kx = mg$
$T = 2\pi\sqrt{\frac{m}{k}}$

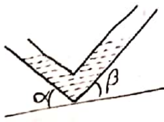


- Two body Oscilla



$F = K(x_1 + x_2)$
$a = \frac{k}{m_1}(x_1 + x_2)$

$\boxed{a = \frac{k}{\mu} x}$ $\boxed{T = 2}$

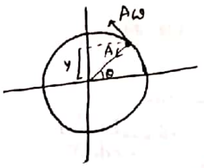


- Physical Pendu

$T = 2\pi\sqrt{\frac{I}{mgl}}$

- $T = 2\pi\sqrt{\frac{l + k^2/l}{g}}$
- $T_{min} = 2\pi\sqrt{\frac{2k}{g}}$

- SHM ↔ Uniform C



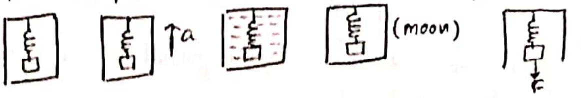
## Method to find T

- Draw the FBD at Equilibrium.
- Displace in slightly from Eqbm.
- Find ~~t~~ net restoring force/torque
- apply $a = \omega^2 x$ or $\alpha = \omega^2 \theta$

## Spring Pendulum

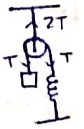
$\rightarrow T = 2\pi\sqrt{\frac{m_{eff}}{K_{eff}}}$

∅ Time period doesn't depend on any external forces.



⇐ All are same

∅



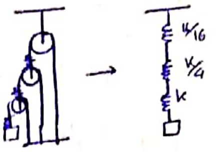
$kx = mg$

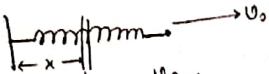
$T = 2\pi\sqrt{m/k}$

### Shortcut

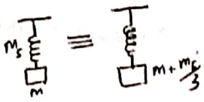
$S_s = \frac{1}{2} S_B$

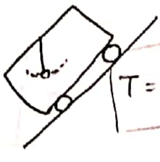
$S_s = n S_B \nearrow k_{av} = n^2 k$



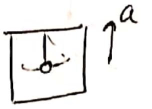


$\hookrightarrow v = \frac{v_0}{l} x$





$$\boxed{T = 2\pi\sqrt{\frac{l}{g\cos\theta}}}$$



$$\boxed{T = 2\pi\sqrt{\frac{l}{g+a}}}$$

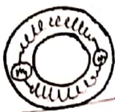
$$\boxed{T = 2\pi\sqrt{\frac{l \cdot \rho_s}{g(\rho_s - \rho_l)}}}$$

## Two body Oscillator

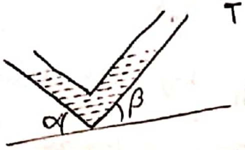
$F = K(x_1 + x_2)$

$a = \frac{k}{m_1}(x_1 + x_2)$

$$\boxed{a = \frac{k}{\mu} x} \quad \boxed{T = 2\pi\sqrt{\frac{\mu}{k}}}$$

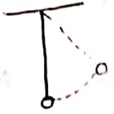


$$\boxed{T = 2\pi\sqrt{\frac{\mu}{k_1 + k_2}}}$$



$$T = 2\pi\sqrt{\frac{l}{g(\sin\alpha + \sin\beta)}}$$

## String Pendulum



$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

- When $\theta_0$ is big.

$$\boxed{T = 2\pi\sqrt{\frac{1}{g}\left(1 + \frac{\theta_0^2}{16}\right)}}$$

- When $l$ is comparable to Radius of Earth

$$\boxed{T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}}$$

## Physical Pendulum

- $T = 2\pi\sqrt{\frac{I}{mgl}}$ ← About AOR; $l$ → dist b/w COM from AOR
- $T = 2\pi\sqrt{\frac{l + k^2/l}{g}}$
- $T_{min} = 2\pi\sqrt{\frac{2k}{g}}$ [when, $k = l$]

## Lissajous figures [Due to superposition of 2SHM]

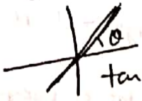
$y = a \sin\omega t$

$x = b \sin(\omega t + \phi)$

**Case-I**

$\phi = 0$

$y = a/b\, x$

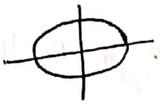


$\tan\theta = a/b$

**Case II**

$\phi = \pi/2$

$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$



**Case III**

If $a = b$

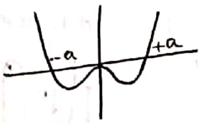
$x^2 + y^2 = a^2$

**Case IV**

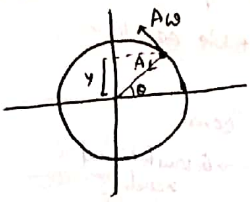
$x = a \sin\omega t$

$y = b \sin 2\omega t$

$y = \frac{2b^2}{a^2}\sqrt{a^2x^2 - x^4}$



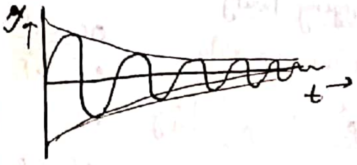
## SHM ⟷ Uniform Circular motion: Phasors

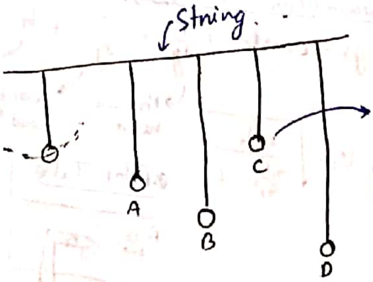


## Damped SHM

$F = -kx - bv$ → Damping co-eff

$A = A_0 e^{-bt/2m} \sin(\omega' t + \phi)$





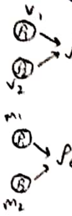
→ C will be in resonance

[All have same frequency]

# Fluids

- Fluids are those which cannot withstand any tangential sheer.
- Relative Density $= \frac{\rho \text{ of the substance}}{\rho \text{ of } H_2O \text{ at } 4°C}$

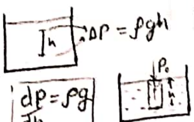
**Finding Eq Density:**



$$\rho_{eq} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

$$\rho_{eq} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

**Variation of pressure w/ Depth**



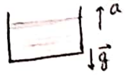
$\Delta P = \rho g h$

$\boxed{\frac{dP}{dh} = \rho g}$

- Absolute P is +ve
- Gauge P can be +ve or −ve.

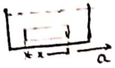
Absolute Pressure $P = P_0 + \rho g h$ → Gauge P
↳ Atm P

**Vertically accelerated**



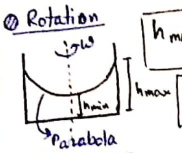
$$\boxed{\frac{dP}{dy} = \rho|\vec{a}_y|}$$

**Horizontally accelerated**



$$\boxed{\frac{dP}{dx} = \rho a_x}$$

**Rotation**

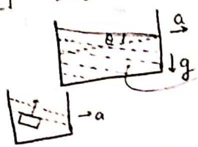


$$\boxed{h_{max} - h_{min} = \frac{\omega^2 x^2}{2g}}$$

$$\boxed{h_{initial} = \frac{h_{max} + h_{min}}{2}}$$

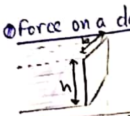
- Liquid can exert force > its weight.

**Angle made by free surface**



$\theta = \tan^{-1}(a/g)$
, lines w/ same P is ∥ to surface.

**Force on a dam**



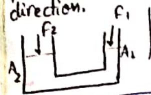
$$\boxed{F = \frac{\rho g b h^2}{2}}$$

$$\boxed{\tau = \frac{\rho g b h^3}{6}}$$

Moment Arm → $h/3$

**Pascal's Law**

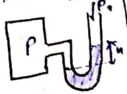
Pressure exerted to any enclosed fuild is transmitted equally and and undiminished to all parts of the fluid in all direction.



$$\boxed{\frac{F_2}{A_2} = \frac{F_1}{A_1}}$$

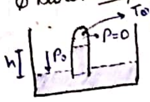
**Pressure Measuring Intruments**

- Manometer



$$\boxed{P - P_0 = \rho g h}$$

- Barometer



$$\boxed{\rho_{Hg} g h = P_0}$$

**Identifying Purity**

$\Rightarrow x(Cu) + (m-x)Au$

$U = \left(\frac{x}{\rho_{Cu}} + \frac{m-x}{\rho_{Au}}\right)\rho_\ell g$

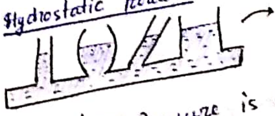
If U is same as $U_{Au}$ (pure) Sample is pure.

**Conditions**

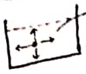
- $\rho_s > \rho_\ell$ [Body will sink]
- $\rho_s = \rho_\ell$ [Equilibrium]
- $\rho_s < \rho_\ell$ [Float partially submerged]

**Hydrostatics** Ideal fluids → • incompressible ($\rho$ is const)
- Non viscous (The layers don't exert any tangential force on each other.
- A liquid is always known by its density.

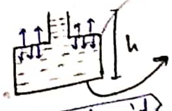
**Hydrostatic Paradox:**



→ Always same height.

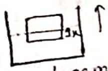


→ Pressure is same is every direction



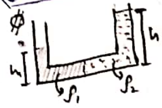
→ This reaction compensates.

$F = \rho g h A$

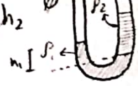


Submerged part remains same if its accelerated

**Multiple Liquid**

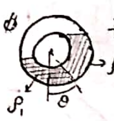


$\rho_1 g h_1 = \rho_2 g h_2$



$\rho_1 g h_1 = \rho_2$

$$x = \frac{(\rho_2 - \rho_1)\ell}{(2\rho_2 - \rho_1 - \rho_3)}$$



$$\tan\theta = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

- when a↑, $h = \frac{h_0 g}{h + a}$
- Air pumped out, h will decrease.
- Air pumped in, h will increase.

**Archemedis Principle**

⌈When a body is partly or fully emerged in a fluid, it experiences a loss in weight i.e. equal to the weight of the liquid that is displaced by the immersed part of the body.⌋

⌈Loss in weight = U = B = wt of liquid displaced⌋

**Centre of buoyancy:** Centre of buoyancy depends on the surface area of the submerged part and it acts through the geometrical centre of the submerged part

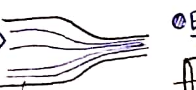
- If Metacentre is above G, it is in stable equilibrium.
- If Metacentre is below G, it is in unstable equilium.

**Barnouli's Theorem (Cons. of Energy)**

$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const.}$ | $\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{const.}$

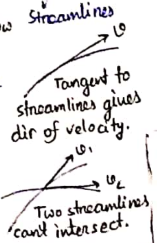
$\frac{P}{\rho g}$ → Pressure head
$\frac{v^2}{2g}$ → Velocity head
$h$ → Gravitational head.

**Hydrodynamics**



- Non viscous
- Incompressible
- Non rotational flow
- Steady flow

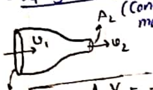
Every point in the flow can be associated w/ a unique velocity, the liquid takes the vel of the point when it reaches there.



Tangent to streamlines gives dir of velocity.

Two streamlines can't intersect.

**Eqⁿ of continuity**



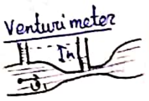
(Cons. of mass)

$\boxed{A_1 V_1 = A_2 V_2 = \cdots}$

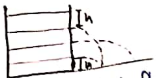
If liq is compressible

$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \cdots}$

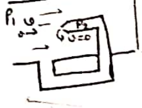
**Venturimeter**



$$\boxed{v_1 = \sqrt{\frac{2gh}{\frac{A_1^2}{A_2^2} - 1}}}$$

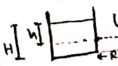


**Pitot Tube**



$$\boxed{v = \sqrt{\frac{2(P_2 - P_1)}{\rho}}}$$

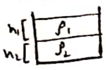
**Velocity of Efflux**



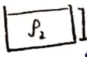
$v = \sqrt{2gh}$

For $R_{max}$, $h = H/2$

**Multiple fluids**

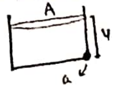


$h' = \frac{\rho_1}{\rho_2} h$



$h_2 + h'$ → $v_{eff} = \sqrt{2g\left(h_2 + \frac{\rho_1}{\rho_2} h_1\right)}$

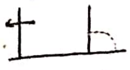
**Time to empty a container**



$$\boxed{t = \frac{A}{a}\sqrt{\frac{2y}{g}}}$$

$\boxed{t^2 \propto y}$

**Thrust Force**



$F_{th} = \rho A v^2$

**Viscosity** ⌈Liquid friction⌉

$F = -\eta A \frac{dv}{dy}$

$\eta$ → coeff of viscosity. → nature of liq, Temp.
$\frac{dv}{dy}$ → vel gradient

- Immisible impurities → viscosity ↓creases.
- Missible Impurities → viscosity ↑creases.

For liquid → $\eta \propto \frac{1}{\sqrt{T}}$ height from base.

For Gas → $\eta \propto \sqrt{T}$

Scalar form → $\boxed{F = -\eta A \frac{\Delta v}{\ell}}$

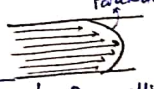
→ Power delivered by Viscous force

$$\boxed{P = \frac{\eta A}{\ell} V_{rel}^2}$$

**Terminal Velocity**

$\boxed{v_t = \frac{2}{9}\frac{R^2 g}{\eta}(\rho_s - \rho_\ell)}$ $\boxed{v_t \propto R^2}$ $\boxed{v_t \propto \frac{1}{\eta}}$

**Tube of flow**



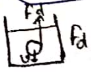
$$\boxed{Q = \frac{dV}{dt} = \frac{\Delta P}{\left(8\eta \ell / \pi R^4\right)}}$$

→ Poiseuilli's Eqⁿ

$Q$ → Hydrodynamic current.

$\Rightarrow \boxed{Q = \frac{\Delta P}{R_h}}$ → Hydrodynamic resistance.

**Stokes Law**

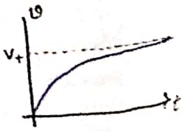


$F_d = 6\pi\eta r v$

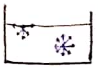
**Motion in viscous fluids**

$a = \frac{mg - U}{m} - \frac{6\pi\eta R v}{m}$
$= \alpha - \beta v$

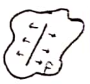
$v = \frac{\alpha}{\beta}(1 - e^{-\beta t})$



**Surface Tension** F
membrane and th



Rain d
are sph



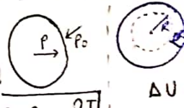
$\boxed{T = F/\ell}$

↳ Electrifica

**Surface Energy** Work

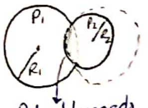
$\boxed{U = T \cdot \text{Area}}$

Excess Pressure in a



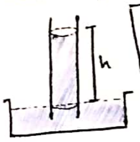
$\boxed{P - P_0 = \frac{2T}{R}}$

Radius of Comm



Bulged towards the bigger one.

**Capillery Tube**



h =

h

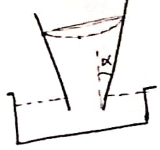
Tube of Insuffici

Liquid will not

It will change its angle.

$\frac{h_0}{\cos\theta_c} = \frac{h}{\cos\theta}$

Slanted Surfac



T

**Surface Tension** Phenomenon due to which the exposed surface of a liquid behaves like a stretched membrane and the liquid tries to minimize it's exposed surface area.

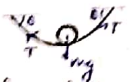
- Rain drops are spherical, for a given volume, the surface area of a sphere is minimum, so raindrops are sperical in shape.
- For a given perimeter, the area of circle is maximum.

$\boxed{T = F/l}$ Force acting per unit length on an imaginary line drawn on the surface of liquid.

→ Temp (T ↑, ST ↓)

→ Impurity → missible (ST ↑) → imissible (ST ↓)

↳ Electrification (ST ↓)



$2Tl\sin\theta = mg$

**Surface Energy** Work done to create new surfaces is stored on the surface of the liquid in form of potential energy.

$\boxed{U = T \cdot \text{Area}}$

**Excess Pressure in a liquid Drop**

$$\boxed{P - P_0 = \frac{2T}{R}}$$

$W = \Delta P 4\pi R^2 \Delta R$

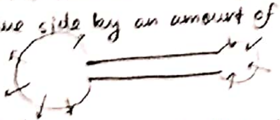
$\Delta U = T\ 8\pi R \Delta R$

**Liquid Bubble**

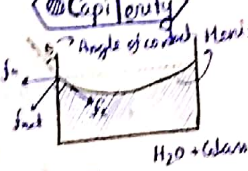
$$\boxed{P - P^\circ = \frac{4T}{R}}$$

$$\boxed{\Delta P \propto 1/R}$$

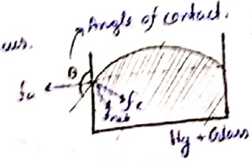
- P on the concave side is greater than that on the concave side by an amount of $2T/R$



**Capilarity**

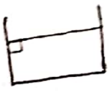


$\lceil f_a > f_c \rfloor$

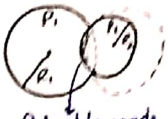


$\lceil f_a < f_c \rfloor$

- Glass - Pure $H_2O$, $\theta_c = 8^\circ \approx 0^\circ$ →
- Glass - Mercury, $\theta_c = 137^\circ \approx 135^\circ$ →
- Ag - Pure $H_2O$, $\theta_c = 90^\circ$ →



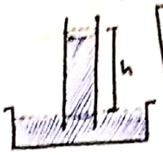
**Radius of Common Interface**



$$\boxed{\frac{1}{R} = \frac{1}{R_1} - \frac{1}{R_2}}$$

Bulged towards the bigger one.

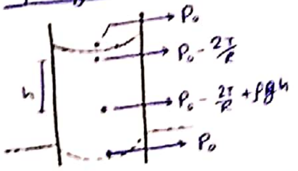
**Capillary Tube**



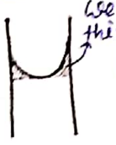
$$\boxed{h = \frac{2T}{\rho g R}}$$ Radius of Menisus

$$\boxed{h = \frac{2T\cos\theta_c}{\rho g r}}$$ Radius of capillary tube

**Pressure Variation in Capillary tube**



**If $\theta_c = 0^\circ$**



we have to consider this part too.

$$\boxed{h = \frac{2T}{\rho g r} - \frac{r}{3}}$$

**Tube of Insufficient height**

⚠ Liquid will not spill out

- It will change its contact angle.

$$\boxed{\frac{h_0}{\cos\theta_c} = \frac{h}{\cos\theta}}$$ New height, New contact angle.

**Energetics**

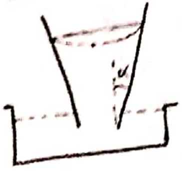
- Work done by Surface Tension → $W_T = T \cdot 2\pi r \times h$
- Change in potential energy, $\Delta U = (\rho\pi r^2 h)\cdot g \cdot h/2$ $[W_T \neq \Delta U]$
- Differnce is desipated as heat, $\boxed{\Delta H = WT - \Delta U}$

**Cylindrical Surface**

$F = \frac{2T}{d}\cos\theta_c \times A$

[as, A (Area of Sheet] is large and d[Seperation) is negligible, F becomes very large]

**Slanted Surface**



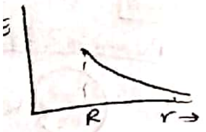
$T \cdot 2\pi r \cos(\theta_c + \alpha) = mg$

Scanned by CamScanner

$(1-\frac{2h}{R})$

SR (behaves like a point mass)

$E = \frac{GM}{r^2}$



perbolic

$\frac{GM}{R^3}(1.5R^2-0.5r^2)$

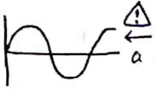
$-\frac{1.5GM}{R}$

# Waves

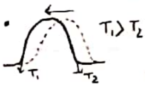
- **Definition:** Any disturbance is space that carries with it momentum and energy.

$$\boxed{\text{Wave}} \rightarrow \text{Matter waves}, \ \text{Electromagnetic}, \ \text{Mechanical} \rightarrow \text{Transverse}, \ \text{Longitudinal.}$$

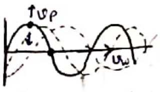
- **Transverse Waves:** Transverse waves can be set up only in bodies fo which Young's Modulus is defind
- Transverse waves can be set up only in solids and surface of liquids.



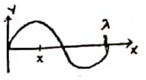
⚠ ← This is not wave motion but a snapshot of wave motion.



$T_1 > T_2$



$v_p \perp v_w$ particle is in SHM.



$y = A\sin(\frac{2\pi}{\cancel{\lambda}}x)$

## General Eqⁿ of waves $[y=f(ax \pm bt)]$

- f must be a function of both x, t
- f must be defined for all position.

$y = f(ax+bt) \rightarrow$ proceeds towards −ve

$y = f(ax-bt) \rightarrow$ proceeds towards +ve.

$\rightarrow$ wave velocity $\boxed{v = \frac{\text{co eff of } t}{\text{co-eff of } x}}$

$y = f(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{T}t)$

$\rightarrow v = \lambda/t \quad \therefore \lambda f = v$

## Plane Progressive Harmonic Wave (PPHW)

- $y = A\sin(kx \pm \omega t)$

k is called wave number

$\rightarrow y = A\sin(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{T}t)$

$\rightarrow y = A\sin 2\pi(x/\lambda \pm t/T)$

$\rightarrow y = A\sin\omega(\frac{k}{\omega}x \pm t)$

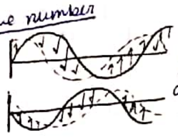
$\rightarrow y = A\sin\omega(x/v \pm t)$

$\rightarrow y = A\sin k(x \pm vt)$

## Phase:

$y_1 = A\sin(kx+\omega t) \rightarrow$

$y_2 = A\sin(\omega t - kx) \rightarrow$



phase differen = π

## Velocity & Acceleration:

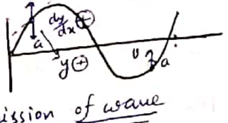
$y = A\sin(kx-\omega t)$

$v_p = dy/dt = -A\omega\cos(kx-\omega t) \ \Big| \ \frac{dy}{dx} = Ak\cos(kx-\omega t)$

$\therefore \frac{dy}{dt}\Big/\frac{dy}{dx} = -\omega/k \quad \therefore \boxed{v_p = -v_w\frac{dx}{dt}}$

$a_p = \frac{d^2y}{dt^2} = -\omega^2 A\sin(kx-\omega t) \ \Big| \ \boxed{a_p = -\omega^2 y}$

$$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$$

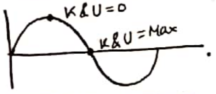


## Energy associated with Wave motion

$K = \frac{1}{2}\mu\omega^2A^2\cos^2(kx-\omega t)$

$U = \frac{1}{2}\mu\omega^2A^2\sin^2(kx-\omega t)$

## Intensity & Power



$\boxed{P = \frac{1}{2}\rho v\omega^2A^2S}$

Energy transmitted per unit time

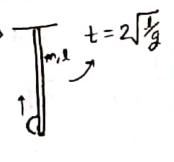
$\boxed{I = \frac{1}{2}\rho v\omega^2A^2}$

Intensity / Energy per unit are per unit time.

## Velocity of transverse wave on a String

$$\boxed{v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\text{Stress}}{\rho}} = \sqrt{\frac{Y\,\text{Strain}}{\rho}}}$$

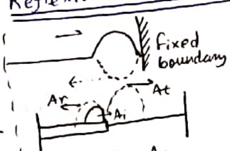
Q»



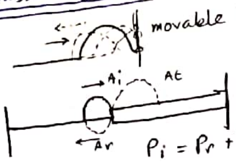
$t = 2\sqrt{l/g}$

- **Dependance:**
- f, T, ω → source
- v → medium
- λ, K → $\lambda = \frac{v}{f}$ → medium, source
- I, loudness → $I \propto A^2$
- phase → Type of Boundary

## Reflexion and Transmission of wave



$A_i + A_r = A_t$



movable boundary.

$P_i = P_r + P_t$

## General differential eqⁿ of wave:

$\frac{d^2y}{dx^2} = v^2\frac{d^2y}{dx^2} \longleftrightarrow \frac{d^2y}{dt^2} = Y/\rho\,\frac{d^2y}{dx^2}$

$\Rightarrow \boxed{v = \sqrt{Y/\rho}} \leftarrow$ for solids

for fluids, $v = \sqrt{B/\rho}$

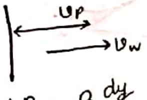
$\rightarrow$ Newton's Formula (Isothermal)

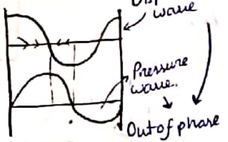
$v = \sqrt{P/\rho}$

$\rightarrow$ Laplace Formula (Adiabatic)

$v = \sqrt{\gamma P/\rho}$

## Longitudinal & Sound Waves





Out of phase by π/2

$\Delta P = -B\frac{dy}{dx}$

$\boxed{\Delta P = P_{max}\sin(kx-\omega t)}$

$\boxed{\Delta P = BAK\sin(kx-\omega t)}$

## Factors affecting velocity of Sound

$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma nRT}{M}} = \sqrt{\frac{\gamma RT}{M_0}}$

- v is independant of P
- $v \propto \sqrt{T}$ [↑ in T, ↑ in v for longitudinal, ↓ in v for transverse]
- $\rho_{dry\ air} > \rho_{moist\ air}$

$v_{dry\ air} < v_{moist\ air}$

- Wind velocity.

$\overset{v_w}{\angle\theta} \rightarrow v_s$

$v_{s/g} = v_s + v_w\cos\theta$

## Relation b/w Intensity and Loudness

$\boxed{\beta = 10\log\frac{I}{I_0}}$

dB

$10^{-12}\,W/m^2$

$$\boxed{I = \frac{\rho v^3 \Delta P_{max}^2}{2B^2}}$$

## Standing Wave:

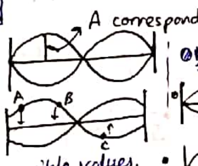
A superposition of two waves such that the amplitude is a function of distance and all the particles execute SHM strarting from the extreme position.

$y_1 = A(kx-\omega t) \rightarrow y = A'\cos\omega t$

$y_2 = A(kx+\omega t) \nearrow \boxed{A' = 2A\cos kx}$

$y_1 = A\sin(kx-\omega t) \rightarrow y = A'\cos\omega t$

$y_2 = A\sin(kx+\omega t) \nearrow \boxed{A' = 2A\sin kx}$



$y = 2A\sin kx\cos(\omega t + \pi/4) \rightarrow$

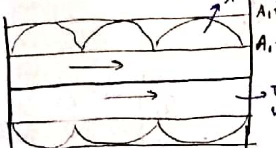
$y = 2A\sin(kx + \pi/6)\cos\omega t \rightarrow$

phase diff (A & B) = 0

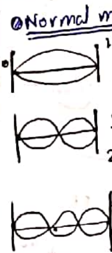
phase diff (A & C) = π } Only possible values.

$\Rightarrow A_1 > A_2$

$y_1 = A_1\sin(kx-\omega t), \ y = A_2\sin(kx-\omega t)$



## Normal modes of vibration

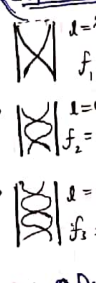


- 1st Harmonic / Fundamental tone: $l = \lambda/2, \ \lambda = 2l, \ f_1 = \frac{v}{2l}$
- $l = \lambda, \ f_2 = v/l$ — 2nd Har / 1st overtone
- $l = \frac{3\lambda}{2}, \ \lambda = \frac{2l}{3}, \ f_3 = \frac{3v}{2l}$ — 3rd Har / 2nd overtone

$$\boxed{f_n = \frac{nv}{2l}}$$

## Open Organ Pipe

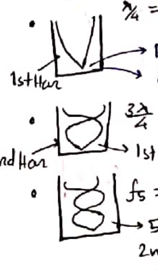
Disp anti, Press node.



- $l = \lambda/2, \ \lambda = 2l$; $f_1 = \frac{v}{2l}$
- $l = \lambda$; $f_2 = v/l$
- $l = \frac{3\lambda}{2}$; $f_3 = \frac{3v}{2l}$

## Closed Organ Pipe

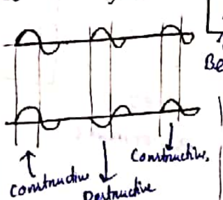
$\lambda/4 = l \quad f = \frac{v}{4l}$



- $\frac{3\lambda}{4} = l \quad f_3 = \frac{3v}{4l}$
- $f_5 = \frac{5v}{4l}$

## Beats

Beasts are a case of super position of time.



$\boxed{\Delta f = f_1 - f_2}$

↑ Beat frequency.

## Waxing

## Waning

## Doppler effect.

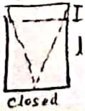
It is the change in frequency heard by an observer due to relative motion b/w the source and observer.

- Motion of obser. changes velocity due to which f changes.
- Motion of source affects the wavelength.

$$\boxed{f = \frac{v - v_0}{v - v_s}f_0}$$

Put $v_0, v, v_s$ with direction sign.

## End correction



$I_e = 0.3d \leftarrow$ Diameter of pipe

$l_{eff} = l + 0.3d$

$l_{eff} = l + 0.6d$

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# KTG

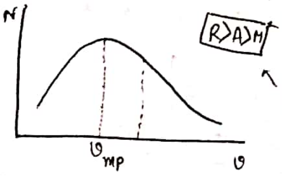
**Gas**:• Ever expanding in nature • Exersts pressure on the walls of the chamber in which it is enclosed.
→ A gas behaves likes an ideal gas at high temperature and low pressure.

**Assumption of KTG** →
- All the molecules are identical and indistinguisable
- The actual volume occupied by the molecules are negligible compared to the volume of the cont
- All the collisions are elastic and the molecules obey newtons laws.

$\boxed{P = \frac{1}{2}\frac{mN}{V}\bar{v}^2}$ $\boxed{P = \frac{1}{3}\rho\bar{v}^2}$ $\boxed{e = \frac{E}{V}}$ $\boxed{E = \frac{1}{2}\frac{M}{V}\bar{v}^2 = \frac{1}{2}\rho\bar{v}^2}$ $\boxed{\frac{P}{E} = \frac{2}{3}}$ $\boxed{v_{rms} = \sqrt{\frac{3RT}{M_0}}}$ $\boxed{\frac{1}{2}m\bar{v}^2 = KE = \frac{3}{2}k_oT}$

• $k_{avg}$ of each molecule $= \frac{3}{2}kT$ • $K_{avg}$ of one mole $= \frac{3}{2}RT$

**Maxwells speed distribution:**



$\boxed{R>A>M}$

$\boxed{v_{mp} = \sqrt{\frac{2RT}{M_0}}}$

$\boxed{v_{avg} = \sqrt{\frac{8RT}{\pi M_0}}}$

$\boxed{v_{rms} = \sqrt{\frac{3RT}{M_0}}}$

**Mean free path** $\boxed{\lambda = \frac{1}{\sqrt{2}(N/V)\pi d^2}}$ → Diameter of each molecule

$\boxed{\lambda \propto \frac{1}{P}}$ $\boxed{\lambda \propto \frac{1}{(N/V)}}$

**Degree of freedom**: It is the number of ways in which a gas can posses energy.

| | MA | D A | Polyatomic non-linear |
|---|---|---|---|
| TKE ($\frac{1}{2}mv^2$) | 3 | 3 | 3 |
| RKE ($\frac{1}{2}I\omega^2$) | 0 | 2 | 3 |
| | 3 | 5 | 6 |
| At High T → | 0 | 2 | 2 |
| | 3 | 7 | 8 |

**Law of Equipartition of Energy**

An ideal gas behaves like an ideal father, Divides among all DOFs equally.

• $\frac{U}{\text{DOF, Mole, Molecule}} = \frac{1}{2}kT$

• $\frac{U}{\text{DOF, Mole}} = \frac{1}{2}RT$ • $\frac{U}{\text{DOF}} = \frac{1}{2}nRT$

• $\boxed{U = \frac{1}{2}nfRT}$

If T = const.
$C = \infty$
$C_{adia} = 0$

Ø There are infinite ways to heat a gas.

$dQ = ncdT \rightarrow C = \frac{dQ}{ndT}$

**Internal Energy**

$\boxed{dU = nC_vdT}$ $\boxed{dU = \frac{n}{2}fRdT}$

**Mayers formula**

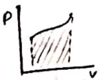
$\boxed{C_p - C_v = R}$ $\boxed{\gamma = 1 + \frac{2}{f}}$

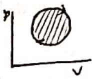
| | MA | D A | PA |
|---|---|---|---|
| $C_v$ | $3R/2$ | $5R/2$ | $3R$ |
| $C_p$ | $5R/2$ | $7R/2$ | $4R$ |
| $\gamma$ | $5/3$ | $7/5$ | $4/3$ |

**$C_p$ & $C_v$ equivalent for mixture of gases**

• $C_{v_{eq}} = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2}$ • $C_{p_{eq}} = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1 + n_2}$

• $\gamma_{eq} = \frac{C_{p_{eq}}}{C_{v_{eq}}}$





**First law of Thermodynamics** [Based on conservation of energy]

$\boxed{dQ = dU + dW}$

$dQ \rightarrow +(\text{heat gained}) - (\text{released})$

$dU \rightarrow +(\uparrow T) - (\downarrow T)$

$dW \rightarrow +(\uparrow V) - (\downarrow V)$

$\boxed{dQ = ncdT}$

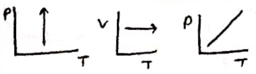
$\boxed{dU = nC_vdT}$

$\boxed{W = \int PdV}$

$\boxed{ncdT = nC_vdT + \int PdV}$

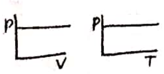
**Iso choric** $[dW = 0]$

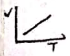
$\boxed{dQ = dU}$ $\boxed{B_{isoch} = \infty}$

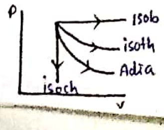


**Isobaric**

$\boxed{C_p = C_v + R}$







**Adiabatic**

$dQ = 0$, $\boxed{dW = -dU}$

$\tan\theta = -\frac{\gamma P}{V}$

$\boxed{PV^\gamma = \text{const}}$

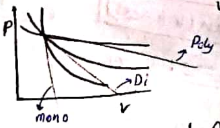
$\boxed{TV^{\gamma-1} = \text{const}}$

$\boxed{P^{1-\gamma}T^\gamma = \text{const}}$

$\boxed{B_{adia} = \gamma P}$

$\boxed{W = \frac{nR(T_f - T_i)}{1-\gamma}}$

Stiffer one is adia.



**Polytropic** (Any other process)

$PV^x = \text{const.}$ $\boxed{W = \frac{nRdT}{1-x}}$

$\boxed{B_{poly} = xP}$

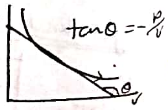
$\boxed{C = C_v + \frac{R}{1-x}}$

$\boxed{C = \frac{R}{\gamma - 1} + \frac{R}{1-x}}$

**Isothermal Process**

$PV = \text{const}$, $\Delta U = 0$

$B_{iso} = P$



$\tan\theta = -\frac{P}{V}$

$\boxed{W = 2.303nRT\log\left(\frac{V_2}{V_1}\right)}$

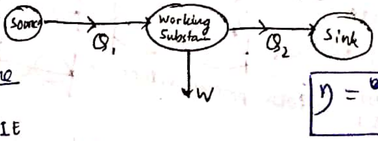
$\boxed{W = 2.303nRT\log\left(\frac{P_1}{P_2}\right)}$

$T_2 > T_1$

**2nd Law of Thermodynamics** (Direction of heat flow)

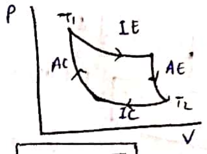
• **Clausius**; It's impossible to transfer heat from a body at low T to a body of High T without help of external agents.

**Heat Engine** -



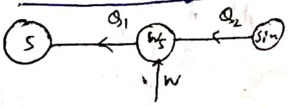
$\boxed{\eta = \frac{Q_1 - Q_2}{Q_1}}$

**Carnot engine**



$\boxed{\eta = 1 - \frac{T_2}{T_1}}$

**Reverse Heat engine**



$\boxed{\beta = \frac{1-\eta}{1-\eta} = \frac{Q_2}{W}}$

Co-eff of performance

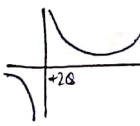
---

$Q_1$

$= \frac{kQ_1Q_2}{r^2}\hat{r} = \frac{1}{4\pi}$

$= \frac{kQ_1Q_2}{r^3}\cdot\vec{r}$

$\boxed{F = 0}$ ← Angular Moment



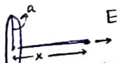
**Field lines**: →

(⁀∿) → can

→ No of field

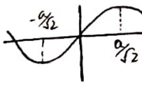
**Special Cases:**

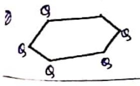
$E_x =$
$E_y =$



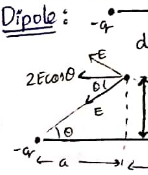
E =

[At centre





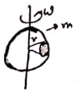
**Dipole**:



$E_{net} = \frac{2kqa}{(r^2+a^2)^{3/2}}$

→ on equitorial line

ipole moment are

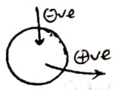


$dF = d$
$F = \omega$

**Gauss Law:**

Flux, $\phi = \int\vec{E}\cdot d$

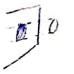
$\phi \propto N$ (No of field through an area



$\oint\vec{E}\cdot d\vec{s} = \frac{q_1 - q_2 + q_3}{\epsilon_0}$

This field is due to charges (inside or

Non conducting sheet

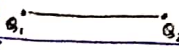


$= \frac{\sigma}{2\epsilon_0}$

Conducting sheet:

$E = \frac{\sigma}{\epsilon_0}$

# Electrostats



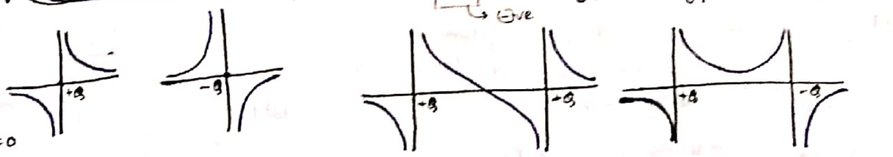
$\boxed{\vec{F} = \frac{kQ_1Q_2}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^2}\hat{r}}$

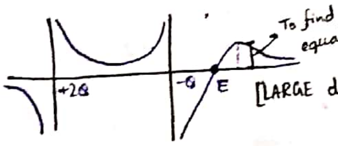
$\boxed{\vec{F} = \frac{kQ_1Q_2}{r^3}\cdot\vec{r}}$ ← central force

$\boxed{\tau_F = 0}$ ← Angular Momentum conserved.

- **Inverse square law**: $\left[F = kr^{n},\ U \propto r^{n+1},\ T \propto r^{|n+1/2|} \middle/ F \propto 1/r^2,\ U \propto 1/r,\ T \propto r^{3/2}\right]$ ↙ This case.
- **Electric field**: $\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \frac{kQ}{r^2}\hat{r}}$ Test charge $\left[F = -dU,\ \boxed{\frac{dF}{dr}} = -\frac{d^2(U)}{dr^2},\ \frac{d^2U}{dr^2} = \oplus ve\right]$ → ⊖ve

→ Graphs:



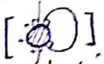


To find this, equate $\frac{dF}{dr} = 0$

[LARGE dist from LARGE charge]

- **Field lines**: → Originates from +ve and ends at ∞ / originates from ∞ and ends at −ve

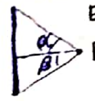
→ can't be electric field lines. cos there are sharp edges.



→ always at 90° to surface

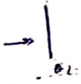
→ No of field lines eminating with a charge is directly proportional to the magnitude of the charge

- **Special Cases**:

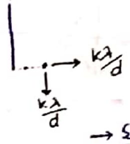


$E_x = \frac{k\lambda}{d}[\sin\alpha + \sin\beta]$

$E_y = \frac{k\lambda}{d}(\cos\beta - \cos\alpha)$



$\alpha = \pi/2$
$\beta = -\theta$



$E_x = \frac{2k\lambda}{d}\ (\alpha=\beta=\frac{\pi}{2})$

$E_y = 0$

→ Sheet (∞)

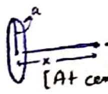
$E = \frac{\sigma}{2\epsilon_0}$

→ Hollow sphere

$\vec{\sigma} = \sigma_0 \cdot \vec{r}$ or

$\sigma = \sigma_0 \cos\theta$ ← azimuthal angle

because ⊕ve in I & IV quad



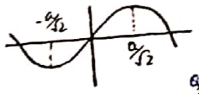
$E = \frac{kxq}{(x^2+r^2)^{3/2}}$

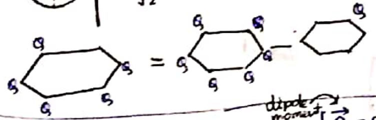
[At centre → E = 0]

$\frac{2k\lambda}{R}\sin\alpha$

→ Disc

$E = \frac{\sigma}{2\epsilon_0}(1-\cos\alpha)$

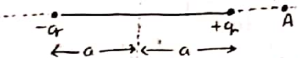




- **Dipole**:

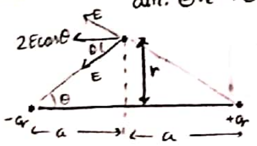
$[\vec{P} = q(2a)]$

dir: ⊖ve → ⊕ve



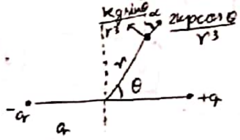
$\vec{E}_A = \frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2} = \frac{4kqar}{(r^2-a^2)^2} = \frac{2kpr}{(r^2-a^2)^2} \approx \frac{2kp}{r^2}$

→ On axial line, electric field and dipole moment are parallel to each othe



$E_{net} = \frac{2kqa}{(r^2+a^2)^{3/2}} \approx \frac{kp}{r^3}$

→ on equitorial line, electric field and dipole moment are anti-parallel.

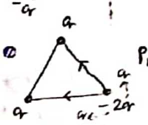


$E_{net} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$, $\tan\alpha = \frac{kp\sin\theta/r^3}{2kp\cos\theta/r^3} = \frac{\tan\theta}{2}$

equivalent to com of half ring.

$P = q\left(\frac{2R}{\pi}\right)$ can be considered

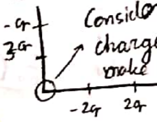
$\frac{2R}{\pi}$ [This half ring can be considered at COM.



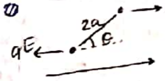
$P_{net} = \sqrt{3}\,p$

$dF = dm\,\omega^2 r$ concept of COM

$F = \omega^2 \int r\,dm$



Consider opposite charges here and make dipoles. (-3q, -q, 2q, -2q)



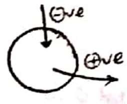
$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

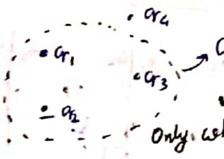
(executes SHM)

- **Gauss Law**:

Flux, $\phi = \int \vec{E}\cdot d\vec{s} = \vec{E}\cdot d\vec{s}$

$\phi \propto N$ (No of field lines passing through an area)





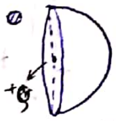
Gaussian surface

$\oint \vec{E}\cdot d\vec{s} = \frac{q_1 - q_2 + q_3}{\epsilon_0}$

↑ This field is due to all the charges (inside or outside)

Only where electric field lines are parallel or perpendicular to the G.S.

[Flux doesn't depend on size or shape of the G.S.]



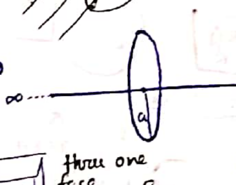
$\phi = \frac{Q}{2\epsilon_0}$

↑ covering half solid angle (Total $\frac{Q}{\epsilon_0}$)

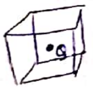
lines from A enters an leaves, so flux ~~can be~~ is zero, not B

E cos45°

$\phi = \frac{1-\cos\alpha}{2}\frac{q}{\epsilon_0}$



$\boxed{\phi_{total} = \frac{\lambda}{\epsilon_0}a}$



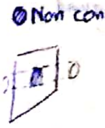
thru one face, $\phi_0 = \frac{q}{6\epsilon_0}$

Flux thru curved surface,

$\phi = \frac{q}{\epsilon_0} - \frac{q}{\epsilon_0}(1-\cos\theta)$

$= \frac{q}{\epsilon_0}\cos\theta$

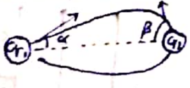
- **Non conducting sheet**:



$E = \frac{\sigma}{2\epsilon_0}$

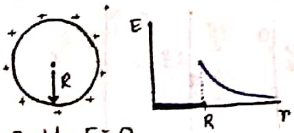
- **Conducting sheet**:

$E = \frac{\sigma}{\epsilon_0}$



$\frac{q_1}{q_2} = \frac{1-\cos\beta}{1-\cos\alpha}$

- **Hollow sphere**:



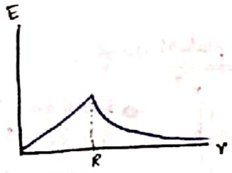
→ Inside, E = 0

→ Outside, it behaves like a point charge

- **Solid nonconducting spheres**

→ Inside, $E = \frac{\rho\vec{r}}{3\epsilon_0}$

→ Outside, behaves like a point charge



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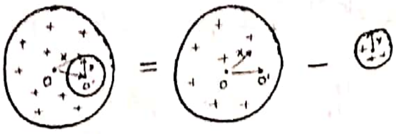
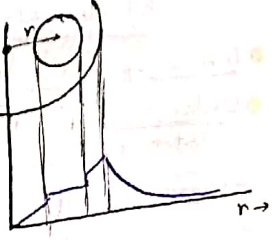
**Cavity:**

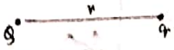


$$\vec{E} = \frac{\rho \vec{x}}{3\epsilon_0} - \frac{\rho \vec{y}}{3\epsilon_0}$$
$$= \frac{\rho}{3\epsilon_0}(\vec{OO'})$$



## Potential Energy

→ Only defined for a system



→ Also known as interraction energy

→ charges are to be put with sign

$F = \frac{kQq}{r^2}$

$dW = \int_{r_i}^{r_f} \frac{kQq}{r^2} dr$

→ Using $r = r_0$ as ref:

$$\boxed{U = kQq\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

$W = -\Delta U$

$U_f - U_i = kQq\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

Taking $r_i \to \infty$

$$\boxed{U = \frac{kQq}{r}}$$

Q>> Potential of a system ~~oo~~

→ Calculate for every ~~posit~~ possible pairs

→ or bring the charges one by one from infinity.

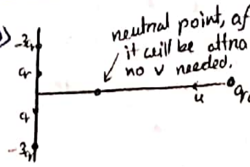
Q>> Closest approach:

fixed → Q

$\frac{kQq}{r} + \frac{1}{2}mu^2 = \frac{kQq}{x}$

→ if not fixed, $\frac{1}{2}\mu v_r^2 = \Delta U$

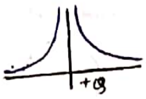
Q>>

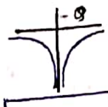


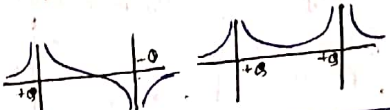
neutral point, after this it will be attracted no v needed.

## Potential

$$\boxed{V = \frac{U}{q} = \frac{kQ}{r}}$$





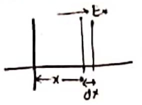


**Relation between field & Potential:** $\boxed{\vec{E} = -\frac{dV}{dr}}$ scalar form $\boxed{\vec{E} = -\frac{\partial}{\partial x}(V)\hat{i} - \frac{\partial}{\partial y}(V)\hat{j} - \frac{\partial}{\partial z}(V)\hat{k}}$

**Tracing Electric field lines:** Q $V = -kxy$, $E_x = ky$, $E_y = kx$, $\frac{dy}{dx} = \frac{x}{y}$, $\int y\,dy = \int x\,dx \Rightarrow y^2 - x^2 = c$

↳ Rect. Hyper.

Q> $V = -ax^2 + b$, $E_x = 2ax$

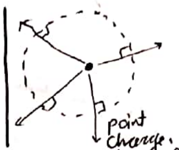


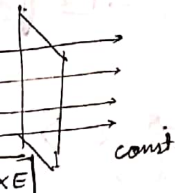
$E_{x+dx}A - E_xA = \frac{dq}{\epsilon_0}$

$2a(x+dx)A - 2axA = \frac{dq}{\epsilon_0}$

$2aAdx = \frac{dq}{\epsilon_0}$

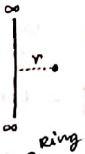
$\rho = 2a\epsilon_0$





**Equipotential Surface:** Allways perpendicular to electric field line. $\boxed{\frac{dV}{dr} \propto E}$

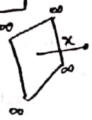
For same chang in potential, $\Delta V$: $\boxed{dr \propto \frac{1}{E}}$



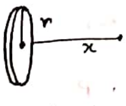
$dV = -\frac{\lambda}{2\pi\epsilon_0}\int \frac{dr}{r}$

$V = \frac{-\lambda}{2\pi\epsilon_0}\ln r + c$ / $V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r}{r_0}$

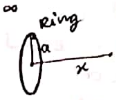
reference point.



$$\boxed{V = \frac{\sigma}{2\epsilon_0}(x - x_0)}$$

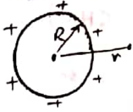


$$\boxed{V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{x^2+R^2} - x\right)}$$



$$\boxed{V = \frac{kq}{\sqrt{x^2+a^2}}}$$

## Potential due to a Hollow charged sphere:

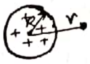


$r = R$ (At surface) $\boxed{V_s = \frac{kQ}{R}}$

$r > R$ $\boxed{V = \frac{kQ}{r}}$ behaves like a point charge.

$r < R$ $\boxed{V = V_s = \frac{kQ}{R}}$

## Potential due to a non-conducting charged sphere:



$r = R$ (At surface) $\boxed{V = \frac{kQ}{r}}$

$r > R$ $\boxed{V = \frac{kQ}{r}}$

$r < R$ $\boxed{V = \frac{\rho}{3\epsilon_0}(1.5R^2 - 0.5r^2)}$

## Lightning Arrester

negatively ionised air attacts the bolts.

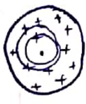
## Self energy:

→ Only defined for charge distribution (not point charge)

$U_s = \int V dq$

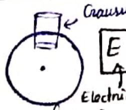
$V = \frac{kq}{r}$

$$\boxed{U_s = 0.5\frac{kQ^2}{R}}$$



$$\boxed{U_s = \frac{0.6kq^2}{R}}$$

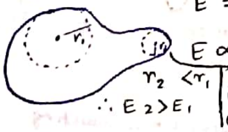
## Behavior of Conductors:



$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

Electric field on surface of conductors.

## Corona Discharge:



$E = \frac{Q}{4\pi\epsilon_0 r^2}$

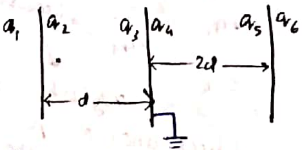
$E \propto \frac{1}{r^2}$

$r_2 < r_1$

$\therefore E_2 > E_1$

E will be very high, ionises sorrounding air, leaks charge

## Different distance

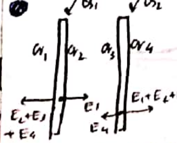


Potential 0 at earthed plate.

$(q_1 + q_2)d + (q_5 + q_6)2d = 0$

## Parallel plates:

charge given,



Total on outer plates = $\Sigma Q$

$q_1 + q_4 = Q_1 + Q_2$

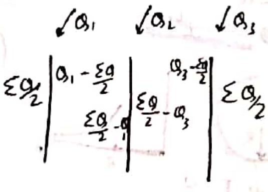
Two adjacent inner plates has equal and opposite charges.

$q_1 + q_2 = Q_1$ ; $q_3 + q_4 = Q_2$

$q_1 = q_2 + q_3 + q_4$

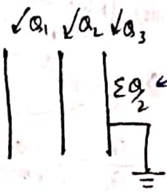
$q_4 = q_1 + q_2 + q_3$

Use these equations



$\frac{\Sigma Q}{2}$ | $Q_1 - \frac{\Sigma Q}{2}$ | $\frac{\Sigma Q}{2} - Q_1$ | $\frac{\Sigma Q}{2} - Q_3$ | $Q_3 - \frac{\Sigma Q}{2}$ | $\frac{\Sigma Q}{2}$

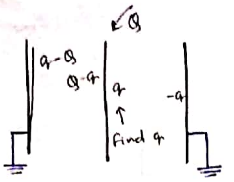
## Earthed



$\frac{\Sigma Q}{2}$ ← This charge will flow to the ground

Scanned by CamScanner

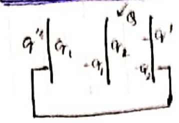
Q>



$Qa = q(a+b)$

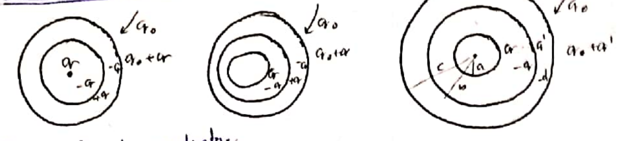
$q = \frac{Qa}{(a+b)}$

### Connecting plates:



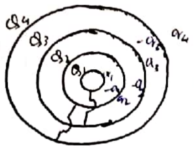
1 Additional eqn: $\boxed{q'' + q_1 + q' \; q_2 = 0}$

### Spherical Conductor:



$$\frac{kq}{a} - \frac{kq}{b} + \frac{kq'}{b} - \frac{kq'}{c} + \frac{k(q_0+q')}{c} = 0$$

### Joining Circular conductors:



$q_1 - q_2 + q_3 = Q_1 + Q_3$ —① Potential on 1st shell

$-q_1 + q_2 - q_3 + q_4 = Q_2 + Q_4$ —②

Potential on 3rd shell.

$$\frac{q_1}{a} - \frac{q_1}{b} + \frac{q_2}{b} - \frac{q_2}{c} + \frac{q_3}{c} - \frac{q_3}{d} + \frac{q_4}{d} =$$

$$\frac{q_1}{c} - \frac{q_1}{c} + \frac{q_2}{c} - \frac{q_2}{c} + \frac{q_3}{c} - \frac{q_3}{d} + \frac{q_4}{d} \quad \text{—(iii)}$$

### Principle of Gauss(?)

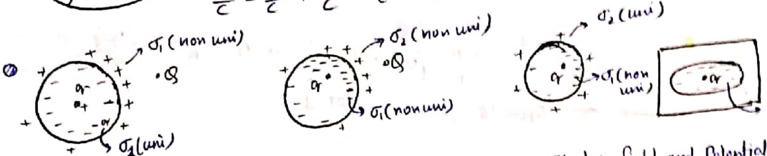
$V_a = \frac{kq_1}{a} + \frac{kq_2}{b}$

$V_b = \frac{kq_1}{b} + \frac{kq_2}{b}$

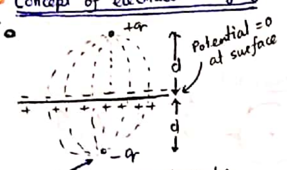
$\Delta V = V_a - V_b = kq_1\left(\frac{1}{a} - \frac{1}{b}\right)$

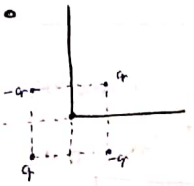
when joined, $\Delta V = 0 \;\therefore\; q_1 = 0 \left[\left(\frac{1}{a} - \frac{1}{b}\right), k \neq 0\right]$

All the charge flows to the outer sphere irrespective of the charges. [Applicable to only 2 isolated spheres.



### Concept of electrical Imaging

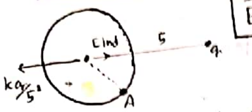




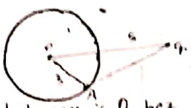
All the charges (induced) can be considered as a opposite sign charge.

### Electric field and Potential due to induced charges:

$\boxed{\vec{E}_{ind} = -\vec{E}_q}$



Q> Find Potential due to induced charge at A



$V_0 = V_q + V_{ind}$ | $V_{ind} = 0$ → Net potential at O due to induced charges is 0, bcz the shell was neutral and dist. form all the charges are same

$V_0 = V_q = \frac{kq}{5}$

At A (surface)

$V_{A/net} = V_{O/net} = \frac{kq}{5}$

$V_{A/net} = V_{A/q} + V_{A/ind} = \frac{kq}{5}$ | $V_{A/q} = \frac{kq}{4}$

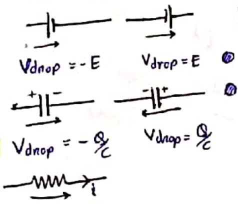
$V_{A/ind} = \frac{kq}{5} - \frac{kq}{4} = \frac{kq}{20}$

---

~~Circuit An~~

## Circuit Analysis with DC

### Kirchoff's Laws:



$V_{drop} = -E$ $\quad V_{drop} = E$

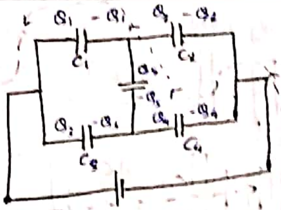
$V_{drop} = -\frac{Q}{C}$ $\quad V_{drop} = \frac{Q}{C}$

$V_{drop} = iR$

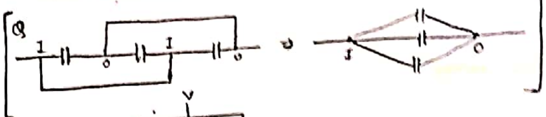
- Both the terminals of the battery supply equal amount of charge.

$\boxed{Q_1 + Q_3 = Q_2 + Q_4}$

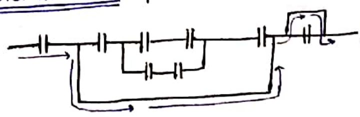
- Total charge on an isolated circuit is zero: $\boxed{\begin{array}{l} -Q_1 + Q_2 + Q_5 = 0 \\ -Q_5 - Q_2 + Q_4 = 0 \end{array}}$
- In any closed loop, the potential supplied by the battery is equal to the potential drop place across the capacitors. $\boxed{-E + \frac{Q_4}{C_4} + \frac{Q_5}{C_5} + \frac{Q_1}{C_1} = 0}$



### Common Potential Method: points with same potential can be joined

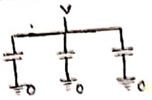


### Shotting:

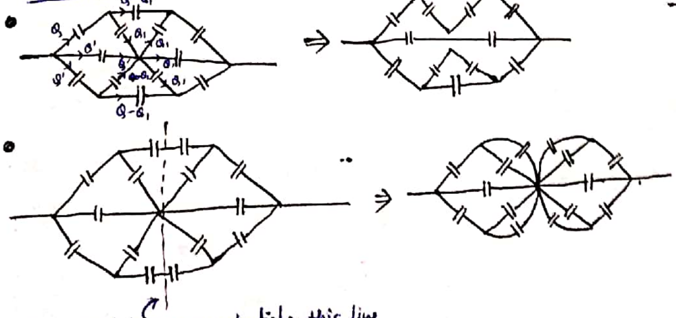


Path with least resistance

### Earthing

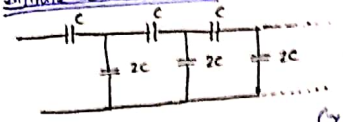


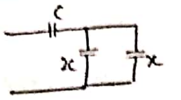
### Connection Removal Method:



Points have same potential on this line

### Infinite Series Problem:



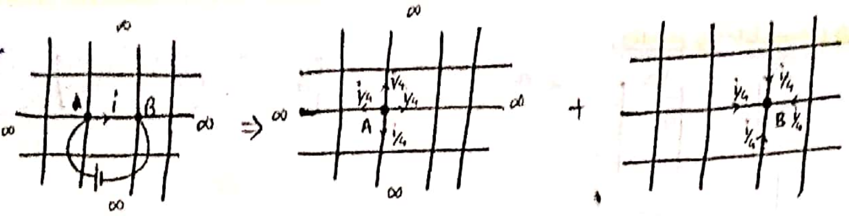


$\frac{(x+2C)C}{2C+x} = x$

→ Solve → $x = -C \pm C\sqrt{3}$

$= C(\sqrt{3}-1)$

Q))

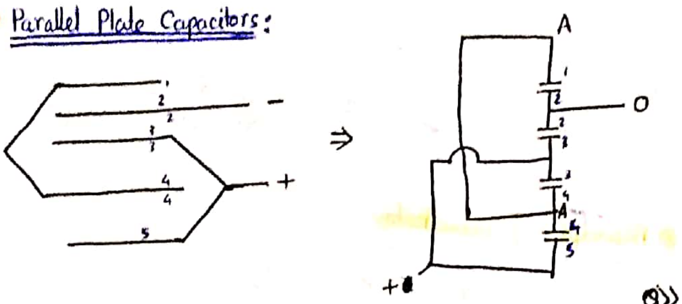


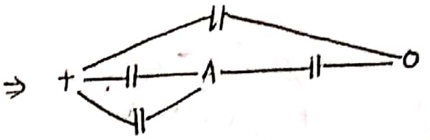
$\therefore i = \frac{i}{4} + \frac{i}{4} = \frac{i}{2}$

Or $E - \frac{i}{2}R = 0$

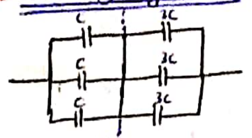
$\therefore \boxed{\frac{E}{i} = \frac{R}{2} = R_{eq}}$

## Parallel Plate Capacitors:





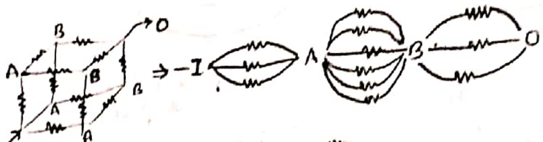
## Method of symmetry:



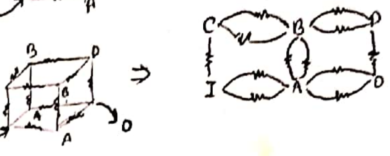
line of symmetry

All points on parallel axis (perpendicular bisector) of a symmetrical circuit are equipotential.

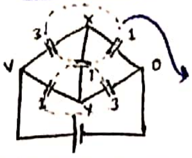
Q))



Q))



## Nodal Analysis:



$3(x-v) + 1(x-y) + 1(x-0) = 0$

$1(y-v) + 1(y-x) + 3(y-0) = 0$

isolated circuit

## Star-Delta Configuration change:

$\Delta$ to $\curlywedge$: $r_1 = \frac{R_2R_3}{\Sigma R}$ [Resistors] $\curlywedge$ to $\Delta$: $R_1 = \frac{\Sigma r_2 r_3}{r_1}$

~~$\Delta$ to $\curlywedge$~~ [Opposite for capacitors]

# Capacitors

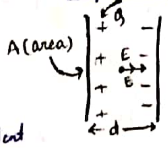
## General Capacitor:

$C = Q/V$

↓

Capacitance, A proportionality constant, Independent of Q or V.

## Capacitance:



$E = \frac{\sigma}{\epsilon_0}$

$\Delta V = Ed$ or $\int \vec{E} \cdot d\vec{r}$

$\Delta V = \frac{\sigma d}{\epsilon_0}$; $C = \frac{QA\epsilon_0}{Qd} = \frac{\epsilon_0 A}{d}$

$\therefore \boxed{C = \frac{\epsilon_0 A}{d}}$

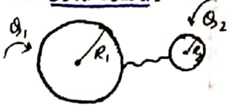
## Isolated spherical capacitors:

$V = \frac{kQ}{R}$

$C = \frac{QR}{kQ} = 4\pi\epsilon_0 R$

$\boxed{C = 4\pi\epsilon_0 R}$

## Connected:

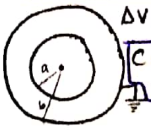


$Q_1 + Q_2 = Q$

$q_1 = \frac{R_1}{R_1+R_2} Q$

$q_2 = \frac{R_2}{R_1+R_2} Q$

## Spherical Capacitor:



$\Delta V = kQ(\frac{1}{a} - \frac{1}{b})$

$\boxed{C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0 ab}{(b-a)}}$

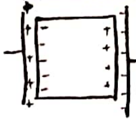
## Cylindrical Capacitors:

$E = \frac{\lambda}{2\pi\epsilon_0 r}$

$\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln b/a$

$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(b/a)}}$

## Effect of Dielectric:



$E_{net} = E_0 - E_{ind}$

$E_{ind} = E_0 - \frac{E_0}{k} = E_0(1 - 1/k)$

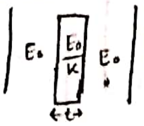
$\boxed{q_{ind} = q(1 - 1/k)}$

## Conducting slab:

$q_{ind} = q(1 - 1/k)$ $k = \infty$, $C = \infty$

$\boxed{q_{ind} = q}$

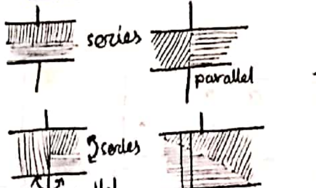
## Partially filled:



$\boxed{C = \frac{A\epsilon_0}{d - t + t/k}}$

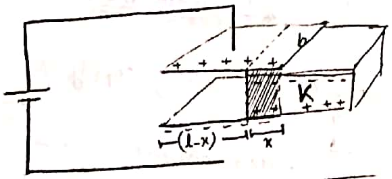
$\boxed{C = \frac{A\epsilon_0}{d - \Sigma t + \frac{\Sigma t}{k}}}$

## Different Combination:



element has segments in series. All elements in parallel.

## Force on A Dielectric:



$C_1 = \frac{bx\epsilon_0 k}{d}$ $C_2 = \frac{b(l-x)\epsilon_0}{d}$

$C_{eq} = \frac{b\epsilon_0}{d}[l + x(k-1)]$

$U = \frac{1}{2}V^2 \frac{b\epsilon_0}{d}[l + x(k-1)]$

$\boxed{F = -\frac{dU}{dx} = -\frac{V_0 b\epsilon_0}{2d}(k-1)}$ → Not SHM but. Periodic

$(l-x) = \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2(l-x)}{a}}$

$\boxed{T = 4t = 4\sqrt{\frac{2(l-x)2md}{V^2 b\epsilon_0(k-1)}}}$

$R_{eq}$

$R_1 = \frac{\Sigma r_2 r_3}{r_1}$

ed:

$Q_1 + Q_2 = Q$

$\frac{R_1}{R_1+R_2} Q$

$\frac{R_1}{R_1+R_2} Q$

onducting slab:

$q_{ind} = q(1-\frac{1}{k})$ $k \to \infty$, $c \to \infty$

$\boxed{q_{ind} = q}$

$\frac{\epsilon_0 k}{d}$ $C_2 = \frac{b(l-x)\epsilon_0}{d}$

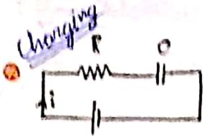
$\frac{\epsilon_0}{d}[l + x(k-1)]$

$\frac{b\epsilon_0}{d}[l + x(k-1)]$

IM but.

# R-C Circuit

Charging



$\tau_c \to$ Transient current [goes thru time lag to attain peak value]

At $t=0$ (when the switch is closed); $i = i_0$ ; $R_{cap} = 0$

At, $t = \infty$ (at steady state): $i = 0$ ; $R_{cap} = \infty$

Kirchoff $\to E - iR - \frac{q}{c} = 0$

$iR = E - \frac{q}{c}$

$iR = \frac{Ec - q}{c}$

$\frac{dq}{dt} = \frac{Ec - q}{RC}$

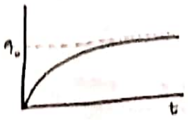
$\int_0^q \frac{dq}{Ec - q} = \int_0^t \frac{dt}{RC}$

$\ln \frac{Ec - q}{Ec} = -\frac{t}{RC}$

$1 - \frac{q}{Ec} = e^{-t/RC}$

$q = EC(1 - e^{-t/RC})$

$\boxed{q = q_0(1 - e^{-t/\tau_c})}$



If $\tau_c = t$

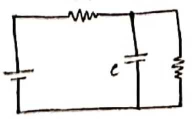
$q = q_0(1 - e^{-1})$

$q = q_0(\frac{e-1}{e})$

$= 0.63 q_0$

$\boxed{= 63\% \, q_0}$

$\boxed{i = i_0 e^{-t/RC}}$

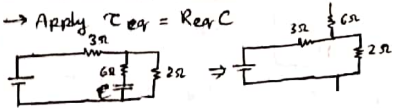


At $t=0$: $i = E/R_1$ ; $V_{drop}^{cap} = iR_2$

At $t=\infty$: $i = \frac{E}{R_1+R_2}$ ; $V_{drop}^{cap} = \frac{ER_2}{R_1+R_2}$ / $Q = \frac{CER_2}{R_1+R_2}$

Methods of find the equivalent time constant:

- → Shot the battery → Make the capacitor an open circuit
- → Find the $R_{eq}$ across the open circuit
- → Apply $\tau_{eq} = R_{eq} C$



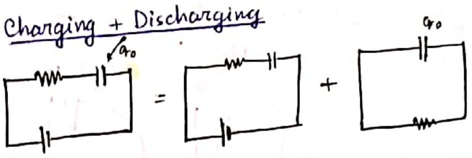
$\Rightarrow R_{eq} = 6R + \frac{6R}{5} = \frac{36R}{5}$

$\therefore \tau_{eq} = R_{eq}C = \frac{36RC}{5}$

Power:

$i = i_0 e^{-t/\tau_c}$ ; $P_{avg} = \frac{\int i^2 R dt}{\int dt} = \frac{\int i^2 e^{-2t/\tau_c} dt}{\int dt}$

Charging + Discharging



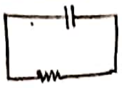
$q_1 = EC(1 - e^{-t/RC})$

$q_2 = q_0 e^{-t/RC}$

$q_{net} = q_1 - q_2$ [As charging and discharging happens in different direction]

$\boxed{q_{net} = EC - (EC + q_0) e^{-t/RC}}$ $\to$ $i = \frac{dq_{net}}{dt}$

Discharging RC circuit:



$\frac{q}{c} - iR = 0$

$\frac{q}{c} = iR$

$-\frac{dq}{dt} = \frac{q}{RC}$

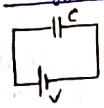
$\boxed{q = q_0 e^{-t/RC}}$

If $t = \tau_c$

$q = \frac{q_0}{e} = 37\%$

Time for it to decrease to 37%.

Energy Stored:

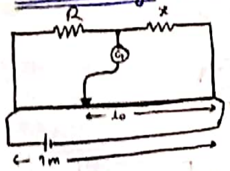


$U = \frac{1}{2}CV^2 \longrightarrow$ Parallel

$= \frac{Q^2}{2C} \longrightarrow$ series

# Electrical Measuring Device

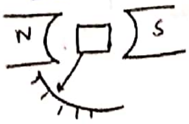
**Meter Bridge:** It works on Wheatstone bridge principle. It is used to determin. the value of an unknown resistance.



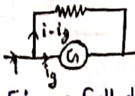
→ when (G) showns no deflection it becomes a balanced WS bridge.

$\frac{R_{wire}}{x} = \frac{1 - l_0}{l_0}$ → Imp ≫ The reading of the meter bridge is most accurate when the balancing point is at the midde of the wire

**Galvanometer:**



**Ammeter:**



→ An ammeters is always connected in series to the register or battery through which the current isto be measured.

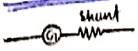
→ An ammeter is, a low resistance instrument.

→ An ideal ammeter has 0 resistance.

→ Galvanometer is converted into an ammeter by connecting a low resistance 'shunt' in parallel with it.

[$i_g$ → Full deflection current ; i = Measured, current]

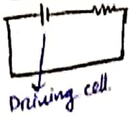
**Voltmeter:** → A voltmeter is connected in parallel to the resistor or battery across which the potential diff is to be measured.



→ Ideal voltmeter has infinite resistance.

→ A galvanometer is converted into a voltmeter by connecting a high resistance 'shunt'.

**Potentiometer:**

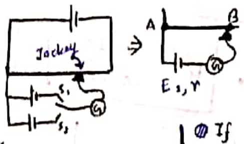


Driving cell.

$V = i \cdot R = i \frac{\rho l}{A} = kl$

$V \propto l \to V/l = $ const.

Potential gradiant



Current will not flow through the lower circuit when potential of A and B are equal.

Step1 ($S_1 \to$ closed, $S_2 \to$ open) : $E_1 = kl_1$

Step 2 ($S_1 \to$ open, $S_2 \to$ closed) : $E_2 = kl_2$

$\frac{E_1}{E_2} = \frac{l_1}{l_2}$

Total Circuit

$I = \frac{E}{R+r'}$

$V_{drop} = ir' = \frac{Er'}{R+r'}$

$V/l = \frac{(\frac{Er'}{R+r'})}{l}$

$E_1 = \frac{V}{l} \times l_1 = (\frac{Er'}{R+r'}) \frac{l_1}{l}$

- If R is increased, r' should be increased to keep V drop const. (J moves right)
- If Ed. is increased, r' should be reduced to keep V drop const. (J moves left)
- If r. is changed, r' should be same.

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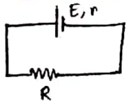
### Measuring Internal Resistance:



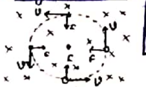
$i = \frac{E}{R+r}$ ; $V = E - iR$

$V = E - \frac{ER}{R+r}$

$\therefore \boxed{r = R\left(E/V - 1\right)}$

---

### Magnetic force:



$\boxed{\begin{aligned}\vec{F} &= q(\vec{v}\times\vec{B})\\ F &= qvB\sin\theta\end{aligned}}$

→ It is a no work force.

[Magnetism]

### Path

If $\theta = \pi/2$

$qvB = \frac{mv^2}{r}$

dependant on $v$

$\boxed{r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}}$

$\omega = v/r = \frac{Bq}{m} = B\alpha$ — specific charge.

$f = \frac{\omega}{2\pi} = \frac{Bq}{2\pi m}$

$T = 1/f = \frac{2\pi m}{qB}$

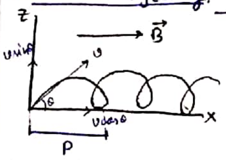
→ Not dependant on $v$ ←

### Undeviated trajectory

Case 1: $\theta = 0°$ or $180°$

Case 2: charge $= 0$

### Deviated Trajectory:



→ After each complete revolution, the charged particle touches the line parallel to ~~the~~ $\vec{B}$ passing through the point of projection.

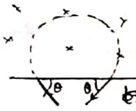
$\boxed{r = \frac{mv\sin\theta}{qB}}$ $\boxed{T = \frac{2\pi m}{qB}}$ $\boxed{P = v\cos\theta \times \frac{2\pi m}{qB}}$

### Deviation of charged particle in the Magnetic field:

If $x > r$ : $\delta = \pi$

If $x \le r$ : $\sin\theta = x/r$

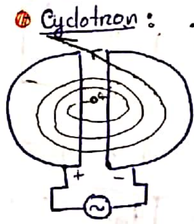
### Time spent by a charged Particle in a magnetic field:



$\boxed{t = \frac{2\pi - 2\theta}{\omega}}$ ~~t = 2π/2~~

$\boxed{t = \frac{2\theta}{\omega}}$

### Cyclotron:



$\boxed{\begin{aligned}T &= \frac{2\pi m}{qB}\\ f &= \frac{qB}{2\pi m}\end{aligned}}$

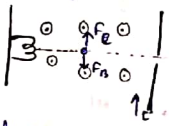
→ Limitations

- Can't be used to accelerate nutral particles
- can't be used to accelerate charged particle will small mass.

### Lorentz force:

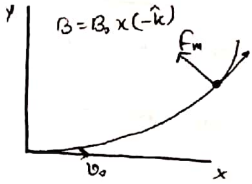
$\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$

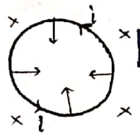
### Velocity selector:



$\boxed{v = E/B}$

### Motion of a charged particle in an non uniform magnetic field:





$\boxed{T = BiR}$

### Force on a current carrying conductor:

One 1 $e^-$; $F_e = e(\vec{v}_d\times\vec{B})$

$F_{tot} = Ne(\vec{v}_d\times\vec{B})$ $[N = nAl]$

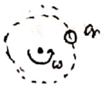
$\boxed{F = i(\vec{l}\times\vec{B})}$ → $\vec{l}$ in direction of current

→ For closed loop ($l = 0$), $\vec{F} = 0$

### Magnetic Dipole:

$M = NiA$ → Area. (N: No of turns)

### Revolving charge as an electric Dipole:



$i = qf = \frac{q\omega}{2\pi}$  $M = i\pi R^2 = \frac{1}{2}q\omega R^2$

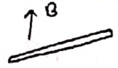
Angular momentum, $L = mR^2\omega$

Gyroscopic constant: $\boxed{M/L = \frac{q}{2m}}$

$\boxed{M/L = \frac{q}{2m}}$

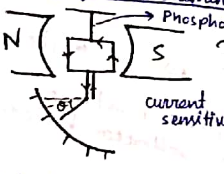
$\boxed{\frac{M}{L} = \frac{q}{2M}}$

### Torque on a current carrying loop in Magnetic field:



$\boxed{\vec{\tau} = \vec{M}\times\vec{B}}$ Direction of $\vec{\tau}$ is always in the dir of AOR. It gives AOR.

### Moving Coil Galvanometer (MCG):

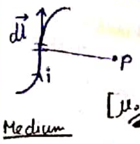


$\tau = NiAB$ , $\tau = k\theta$

current sensitivity → $\boxed{\frac{\theta}{i} = \frac{NAB}{k}}$

$\boxed{\theta/V = \frac{NAB}{kR}}$ → Voltage Sensitivity

### Biot-Savart law:



Free space: $dB = \frac{\mu_0}{4\pi}\cdot\frac{idl\sin\theta}{r^2}$

$[\mu_0/4\pi = 10^{-7}]$

Medium

$dB_{med} = \frac{\mu_0\mu_r}{4\pi}\cdot\frac{idl\sin\theta}{r^2}$

Vector form

$d\vec{B} = \frac{\mu_0}{4\pi}\cdot i\,\frac{\vec{dl}\cdot\vec{r}}{r^3}$

### Finite wire:

$\boxed{B = \frac{\mu_0 i}{4\pi d}(\sin\alpha + \sin\beta)}$

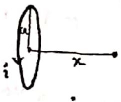
### Infinite:

$\boxed{B = \frac{\mu_0 i}{2\pi d}}$

### Semi-Infinite:

$\boxed{B = \frac{\mu_0 i}{4\pi d}}$

### Circular loop:



$\boxed{B = \frac{\mu_0 ia^2}{2(a^2+x^2)^{3/2}}}$ $\boxed{B = \frac{\mu_0 i}{2r}}$

$\boxed{B = \frac{\mu_0 i}{4\pi r}\cdot\theta}$

### Magnetic field due to a Solenoid :

$B = \frac{\mu_0 n i}{2}(\cos\alpha - \cos\beta)$

For a solenoid : $B = \mu_0 n i$

At end : $B = \frac{\mu_0 n i}{2}$

### Infinite sheet :

charge density

$\boxed{B = \frac{\mu_0 j d x}{2\pi r}}$ $\boxed{B = \frac{\mu_0 j}{2}}$

→ Is independant of dist from it.

### Ampere's Circuital Law :

$\oint \vec{B} \cdot d\vec{s} = 0$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enclosed}$

Amperean loop

only enclosed current

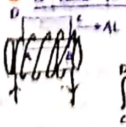
⊙ $i_1$ ⊙ $i_3$ $\oint \vec{B} \cdot d\vec{l} = (i_1 + i_3 - i_2)\mu_0$

⊗ $i_2$ ; due to all changed inside or outside.

⊗ $i_4$

→ Amperean loop should be ∥ or ⊥ to the magnetic field.

### B due to solenoid using ACL



AL

$\oint_A^A \vec{B}\, d\vec{l}$ & $\int_D^A \vec{B} \cdot d\vec{l} = 0$ ← ⊥ to B

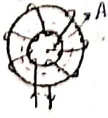
$\int_C^D \vec{B} \cdot d\vec{l} = 0$ ← outside $\int_A^B \vec{B} \cdot d\vec{l} = Bl$

$\therefore \oint \vec{B} \cdot d\vec{l} = Bl \rightarrow Bl = \mu_0 (n i l)$

Used to produce uniform circular magnetic field,

$\boxed{B = \mu_0 n i}$

### Toroid using ACL



AL $\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r$ , $i_{enc} = Ni$

$B \cdot 2\pi r = \mu_0 Ni$

$\boxed{B = \frac{\mu_0 Ni}{2\pi R}}$

### Magnetic field inside a current carrying conductor :

At $\boxed{B = \frac{\mu_0 i}{2\pi r}}$ ,

At $\boxed{B = \frac{\mu_0}{2} j r}$

### Cavity

$B_1 = \frac{\mu_0 j}{2}x - \frac{\mu_0 j}{2}y$

$= \frac{\mu_0 j}{2}(x_1 - y_1)$

$= \frac{\mu_0 j}{2}(OO')$ [⊥ to OO']

→ Magnetic field inside a cavity is constant and only depends on the distance the centres

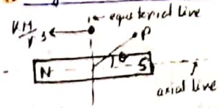
### Force between to parallel current carrying conductors :

$B = \frac{\mu_0 i}{2\pi d}$ , $F = i_2(\vec{l} \times \vec{B}) = \frac{i_2 l \mu_0 i_1}{2\pi d}$

$\boxed{\frac{f}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}}$

Doesn't obey Newton's 3rd Law.

### Magnetic field by Bar magnet:



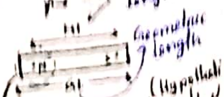
[$\vec{B}$ is ∥ to $\vec{M}$]

At point $P_1$, $B = \frac{KM}{r^3}\sqrt{1 + 3\cos^2\theta}$

→ Magnetic field lines make closed loop

→ Magnetic field lines are conservative

### Pole magnet

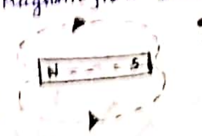


$\boxed{\frac{M}{Al} = 0.84}$

(Hypothetical)

M → monopole strength ↙

$\boxed{M = ml}$ (Dir: S to N)



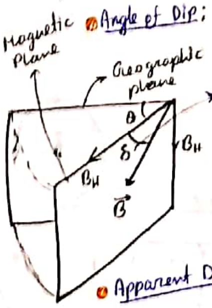
$M' = m \times 2R\sin 30°$

Torque, $\vec{\tau} = \vec{M} \times \vec{B}$

Potential Energy, $U = -\vec{M} \cdot \vec{B}$

Work done, $W = MB(\cos\theta_1 - \cos\theta_2)$

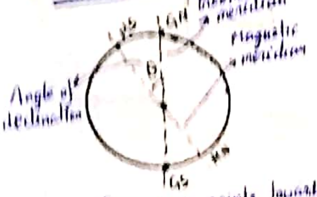
### Angle of Dip;



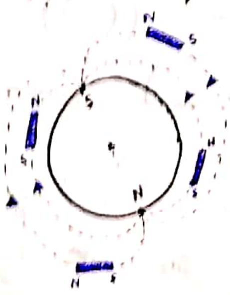
→ Angle of Dip/Angle of inclination (Angle b/w total magnetic field and horizontal)

$\boxed{\tan\delta = \frac{B_V}{B_H}}$ / $\boxed{B = \sqrt{B_H^2 + B_V^2}}$

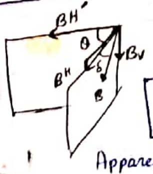
### Earth's Magnetism;



North of compass points towards magnetic south and Geometric N.



### Apparent Dip:



Angle of dip measured in geographical plane.

$\boxed{\tan\delta' = \frac{B\sin\delta}{B\cos\delta\cos\theta} = \frac{\tan\delta}{\cos\theta}}$

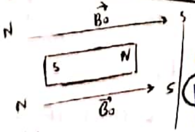
Apparent Dip.

### Substances :

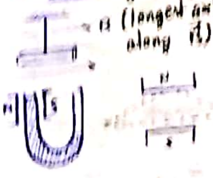
→ Para magnetic [e.g. $O_2$, Al, Mn]

→ Diamagnetic [e.g. Cu, Au, $H_2O$, Alcohol]

→ Ferromagnetic [e.g. Fe, Ni, Co]



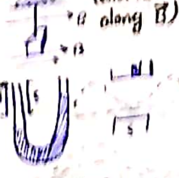
### Para : gets alligned (longest axis along B)



Para

(P: Para/P: Pass large)

### Dia : gets alligned (shortest axis along B)



Dia

(D: Min/Dia: Diverge)

### Ferro : (Nothing))

### Intensity of Magnetisation :

$\boxed{I = \frac{M}{V}}$ → To which extent the object can be magnetised/

Magnetic moment gained per unit volume.

### Magnetic Susceptibility;

$\boxed{\chi = \frac{I}{H}}$

$\chi > 0$ → Para

$\chi < 0$ → Dia

$\chi \gg 0$ → Ferro

Relative permeability

$\boxed{\mu_r = 1 + \chi}$

### Magnetising force :

$\boxed{H = \frac{B}{\mu}}$ How much the field can magnetise

→ Net Magnetic field:

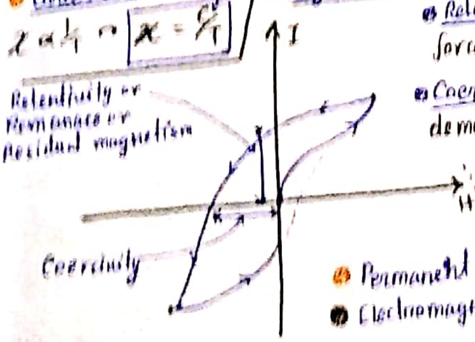
$\boxed{\frac{B}{\mu} = \frac{B_0}{\mu_0} - I}$

### Curie's Law:

Curie's Constant

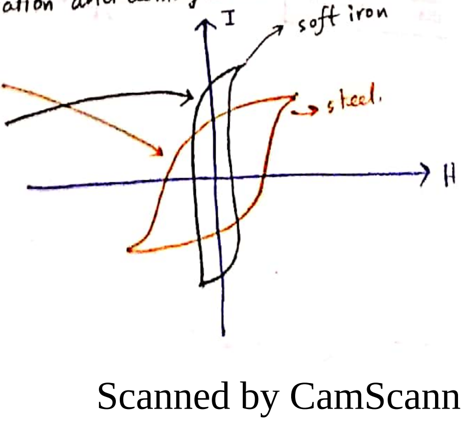
$\chi \propto \frac{1}{T}$ ⤳ $\boxed{\chi = \frac{C}{T}}$

### Hysteresis Loop:



- Retentivity: It is the magnetism retained by the substance when the magnetisin force is brought down to 0.
- Coercivity: It is the reverse magnetic field that has to be applied to completel demagnetise the substanke.
- Area under the Hysteresis curve is directly proportional to the energy loss taking place during magnetisation and demagnetisation

- Permanent Magnetic substance: High coercivity
- Electromagnetic substance: High initial I and low coercivity.



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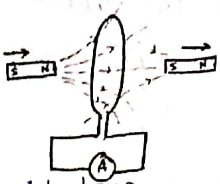
# EMI

**Faraday's Law:** Whenever the flux linked with a conducting loop changes an emf is induced in the loop whic lasts as long as the change takes place.



$\boxed{\phi = \int \vec{B} \cdot d\vec{s}}$, B is non uniform / $\boxed{\phi = \vec{B} \cdot \vec{s}}$, B is uniform / $\phi$ → SI unit = Weber

Area vector

flux

Induced EMF

due to Lenz's Law

~~Lenz's Law~~ Induced EMF: $\phi = BA\cos\theta$ , $\boxed{e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\theta)}$

Induced charge

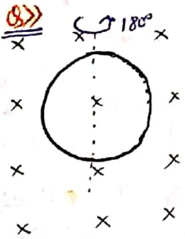
$\boxed{i = \frac{e}{r} = -\frac{d\theta}{Rdt}}$

Induced current

$\boxed{q = -\frac{\Delta\phi}{R}}$

- Induced emf and current is time dependant
- charge flowing is independant of time

Q>>



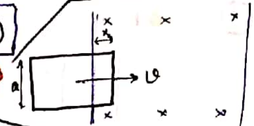
$\phi_i = B\pi R^2 \cos\theta$

$\phi_f = B\pi R^2 \cos 180°$

$\Delta\phi = 2B\pi r^2$

**Ways to induce emf:** $\boxed{e = -\frac{d}{dt}(BA\cos\theta)}$

→ By varying B; → By varying area: Q>>

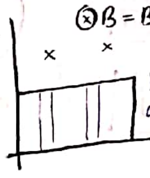


$t = \frac{a}{v}$

$\phi = Bax$

$e = -\frac{d\phi}{dt} = -Bav$

Q>>



$⊗ B = B_0 x(-\hat{k})$

no of field lines in each particular area is constant.

$\therefore \boxed{e = 0}$

Q>>

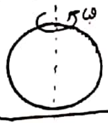
$\phi = \frac{B\pi r^2}{2\pi}\theta = \frac{Br^2\theta}{2}$

$e = \frac{Br^2}{2}\frac{d\theta}{dt} = \frac{Br^2}{2}\omega$

$\frac{d\omega}{dt} = \alpha$

→ By varying θ: $\phi = BA\cos\omega t$

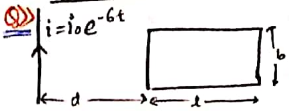
Q>> ⊗



$e = \omega BA \sin\omega t$

→ By varying i that produces B:

Q>>



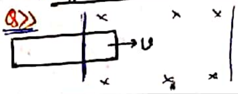
$i = i_0 e^{-6t}$

$\phi = \frac{\mu_0 i_0 e^{-6t} b}{2\pi} \ln\left(\frac{d+l}{d}\right)$

$e = -\frac{d\phi}{dt}$

→ By varying two quantities:

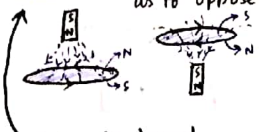
Q>>

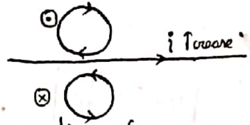


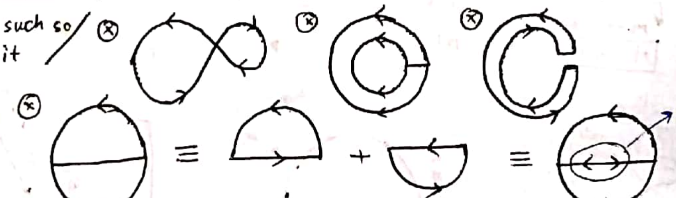
$B = B_0 t$

$e = -\frac{d}{dt}(BA) = -\left\{B\frac{dA}{dt} + A\frac{dB}{dt}\right\}$

**Lenz's Law:** The direction of the ~~induced~~ induced emf is such so as to oppose the cause that produces it



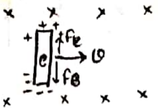




Cancel out.

Lenz's Law is based on conservation of energy

**Motional EMF:**



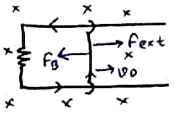
Transfer will take place until.

$eVB = eE$

$\boxed{VB = E}$ Motional EMF

$\boxed{\text{Induced EMF} = El = VBl}$

Q>>

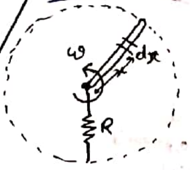


$i = \frac{Blv_0}{R}$

$F_{ex} = F_B = Bil$

$\boxed{F_{ex} = \frac{B^2 l^2 v_0}{R}}$

Q>>

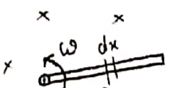


$i = \frac{1}{2}\frac{B\omega l^2}{R} = \frac{B\omega l^2}{2R}$

$d\tau = \frac{B^2\omega l^2}{2R} \cdot dx$

$\tau = \frac{B^2\omega l^2}{2R}\int_0^l x\,dx = \frac{B^2\omega l^4}{4R}$

**EMF Induced in a Rotating Rod:**



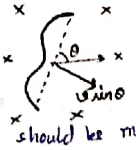
$e = Blv$

$\int de = \int B dx\, \omega x$

$e = \frac{1}{2}B\omega l^2$

→ The EMF induced in a rod about a point perpendicular to the length of the rod is const. and is equal to the EMF abt. its end.

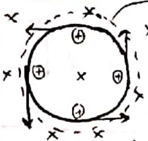
→ Curved Conductor:



$e = \vec{l} \cdot (\vec{v} \times \vec{B})$

→ For EMF to be produced, $\vec{l}, \vec{v}, \vec{B}$ should be mutually ⊥.

**Induced Electric field:** $(B = B_0 t)$



→ Time varying magnetic field gives rise to an electric field known as induced electric field.

→ This electric field is non-conservative in nature.

→ This electric field forms closed loops

→ The direction of force due to this electric field is tangential to the loop.

→ The presence of elec.field. doesn't depend on the presence of conducting loop The conducting loop is used to only test the presence of elec. field.

**Faraday's Law:**

$e = -\frac{d\phi}{dt}$ $dV = -\int \vec{E} \cdot d\vec{l}$

$\boxed{\frac{d\phi}{dt} = \int \vec{E} \cdot d\vec{l}}$

**Switching Effect:** (Magnetic field switched off after time 't'.

$B_0$ $E \cdot 2\pi r = \pi r^2 \cdot \frac{B_0}{t}$

$E = \frac{B_0 r}{2t}$ , $\tau = Eqr = \frac{B_0 q r^2}{2t}$

$\frac{B_0 q r^2}{2t} = I\alpha$

Self Induct

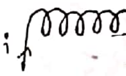
N

$\boxed{L = \frac{N\phi}{i}}$

$e = \frac{-d(N}{dt}$

Methods to

- Give a cur
- Find the m
- Apply $\frac{N\phi}{i}$



Energy st

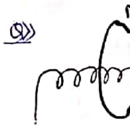
$e = -L\frac{di}{dt}$

$\frac{dW}{dt} = -Li$

$U = -dW$

$\boxed{U = \frac{Li^2}{2}}$

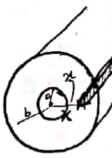
Q>>



$B = \frac{\mu_0 N}{l}$

M =

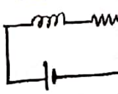
Q>> Co-axial



$\phi = \frac{\mu_0 i l}{2\pi}$

L =

**LR Circui**



$E - iR - L$

$\frac{di}{dt} = E-$

$\int_0^i \frac{di}{E - iR} = \int$

# Inductance

- **Self Induction:** It is the production of emf in a coil due to change in current in same coil

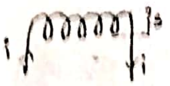
$N\phi \propto i \Rightarrow N\phi = Li$ → Self inductance

$\boxed{L = \frac{N\phi}{i}}$ depends on: → shape and size of loop.
→ No of turns
→ Medium

$$\boxed{e = -\frac{d(N\phi)}{dt} = -L\frac{di}{dt}}$$

- **Methods to find the Self Inductance:**

→ Give a current I to the inductor
→ Find the magnetic field linked w/ the inductor
→ Apply $\frac{N\phi}{i}$



$B = \frac{\mu_0 N i}{l}$

$\phi = \frac{\mu_0 N i s}{l}$

$L = \frac{\mu_0 N^2 s}{l}$

$\boxed{L \propto N^2}$

- **Energy stored in an Inductor:**

$e = -L\frac{di}{dt}$, $P = ei = -Li\frac{di}{dt}$

$\frac{dW}{dt} = -Li\frac{di}{dt} \Rightarrow dW = -Lidi$

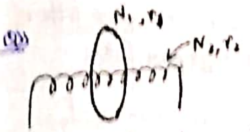
$\Rightarrow U = -dW = L\int idi$

$$\boxed{U = \frac{Li^2}{2}}$$

- **Energy Density:**

$$\boxed{u = \frac{U}{volume} = \frac{1}{2}\frac{B_0^2}{\mu_0}}$$

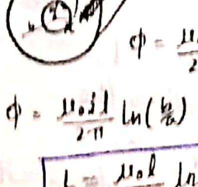
(1)



$B = \frac{\mu_0 N_2 i}{l} \rightarrow \phi_{coil} = \frac{\mu_0 N_2 i}{l} \times \pi r_1^2$

$M = \frac{N_1 \phi_{coil}}{i} = \frac{\mu_0 N_1 N_2 i \pi r^2}{l}$

(3) Co-axial cable:



$B = \frac{\mu_0 i}{2\pi x}$

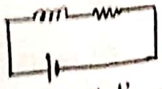
$d\phi = \frac{\mu_0 i}{2\pi x} \cdot l dx$

$\phi = \frac{\mu_0 i l}{2\pi}\int_a^b \frac{dx}{x}$

$\phi = \frac{\mu_0 i l}{2\pi}\ln\left(\frac{b}{a}\right)$

$$\boxed{L = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right)}$$

- **LR Circuits:**



$E - iR - L\frac{di}{dt} = 0$

$\frac{di}{dt} = \frac{E - iR}{L}$

$\int_0^i \frac{di}{E - iR} = \int \frac{dt}{L}$

$-\frac{\ln(E - iR)^i}{R} = \frac{t}{L}$

$\ln\frac{E - iR}{E} = -\frac{Rt}{L}$

$1 - \frac{iR}{E} = e^{-\frac{Rt}{L}}$

$i = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$

~~t = i~~

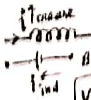
$$\boxed{i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)}$$

$$\boxed{\tau_L = L/R}$$

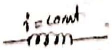
★ At t = 0
$i = 0$, $R_L = \infty$

★ At t = ∞
$i = i_0$
$R_L = 0$

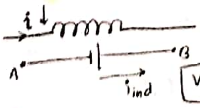
- **Inductor:** ( ) → Inductor opposes the change of current.



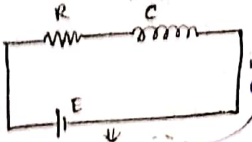
$\boxed{V_A - V_B = L\frac{di}{dt}}$



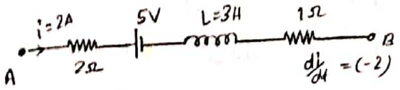
$\boxed{i_{ind} = 0}$



$\boxed{V_A - V_B = -L\frac{di}{dt}}$



$E - iR - L\frac{di}{dt} = 0$ ← is to be put w/ sign



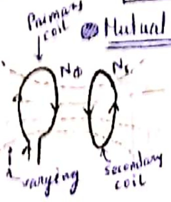
$V_A - 2 - 5 - 3(-2) - 1 = V_B$

- **Self inductance of a Donut shaped toroid:**

$B = \frac{\mu_0 N i}{2\pi x}$ $d\phi = \frac{\mu_0 N i}{2\pi x} \cdot h dx$

$\int d\phi = \frac{\mu_0 N i h}{2\pi}\int_a^b \frac{dx}{x}$ → $\phi = \frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right)$, $L = \frac{N\phi}{i}$

$L = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$



- **Mutual Inductance:** → Mutual Induction cannot occur without self inductance.

$N_s\phi_s \propto i_p$

$N_s\phi_s = Mi_p$

Mutual inductance.
Depends on → Shape & Size → Medium → No of turns
→ Relative orientation of two coils

$\boxed{M = \frac{N_s\phi_s}{i_p}}$ → current in primary coil

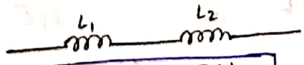
- **Method to find Mutual inductance:**

→ Give current to primary → Find B due to primary
→ Find φ linked with secondary. → $\boxed{M = \frac{N_s\phi_s}{i_p}}$

- **Real condition (Coupling):** $M_{12} = K\frac{N_2\phi_2}{i_1}$ $M_{21} = K_2\frac{N_1\phi_1}{i_2}$

$M^2 = K_1K_2\frac{N_2\phi_2}{i_1} \times \frac{N_1\phi_1}{i_2} = L_1L_2K_1K_2$

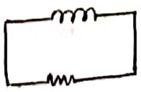
$M = \sqrt{K_1K_2}\sqrt{L_1L_2} = K\sqrt{L_1L_2}$ → coupling
↳ co-eff of coupling



$\boxed{L_{eq} = L_1 + L_2 + 2M_{12}}$

$\boxed{L_{eq} = L_1 + L_2 - 2M_{12}}$

- **Discharging:** $-L\frac{di}{dt} - iR = 0 \Rightarrow \frac{di}{dt} = -\frac{iR}{L}$

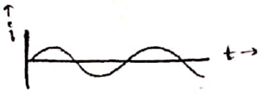


$\Rightarrow \int_0^i \frac{di}{i} = -\frac{R}{L}\int_0^t dt$

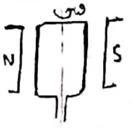
$\Rightarrow \ln(i/i_0) = -\frac{Rt}{L}$

$$\Rightarrow \boxed{i = i_0 e^{-\frac{Rt}{L}}}$$

# AC.



$$\boxed{i = i_0 \sin(\omega t + \phi)}$$



$\phi = BA\cos\omega t$

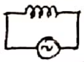
$i = i_0 \sin\omega t$

- Charge flown in full cycle = 0
- $\omega = 2\pi f$, $f = \frac{\omega}{2\pi}$ → 50 Hz in IND
- Charge flown in Half cycle,

$$\boxed{\langle i\rangle = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} = 0.637\, i_0}$$

DC equivalent of AC in terms of charge flown.

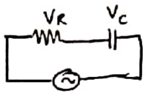
## Purely inductive circuit



$v = L\frac{di}{dt} = L i_0\omega \cos\omega t$

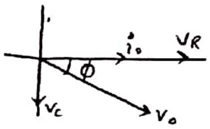
$$\boxed{\begin{aligned} v &= i_0(\omega L)\sin(\omega t + \tfrac{\pi}{2}) \\ V &= i(X_L)\sin(\omega t + \tfrac{\pi}{2}) \end{aligned}}$$

Inductive reactance



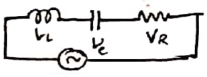
$V_0 = \sqrt{V_C^2 + V_R^2}$ $\tan\phi = \frac{X_C}{R}$

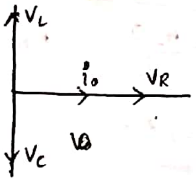
$Z = \sqrt{R^2 + X_C^2}$



Current leads the voltage by $\phi$ i.e. $\tan^{-1}\left(\frac{X_C}{R}\right)$

## LCR in Series:





$V_0 = \hat{i}V_R + \hat{j}(V_L - V_C)$

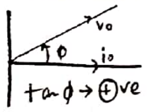
$i_0 Z = i_0 R + j(iX_L - iX_C)$

$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

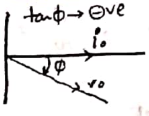
Total impedance

$Z = \sqrt{R^2 + (X_L - X_C)^2}$,

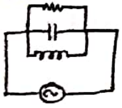
→ If $X_L > X_C$

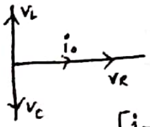


→ If $X_L < X_C$



## LCR in Parallel:





$[j = \sqrt{-1}]$

Admittance

$Y = \frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{(-j)X_C}$

$= \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$

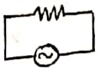
$|Y| = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$

## RMS value:

$i_{rms} = \sqrt{\frac{\int i_0^2 \sin^2\omega t\, dt}{\int dt}}$ effective value of AC

Full cycle → $\boxed{i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707\, i_0}$

## Purely resistive circuits



$v = iR = i_0 R\sin\omega t$

## Purely capacitive Circuits

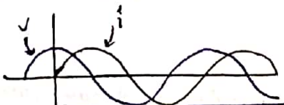
$V = V_0\sin\omega t$

$q = CV_0\sin\omega t$

$i = \frac{dq}{dt} = C\omega\, V_0\cos\omega t$

$\boxed{X_C = \frac{1}{C\omega}}$ $= \frac{V_0}{\left(\frac{1}{C\omega}\right)}\cos\omega t = \frac{V_0}{\frac{1}{C\omega}}\sin(\omega t + \frac{\pi}{2})$

→ Capacitor reactance [R offered by the C to AC]

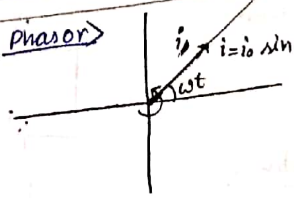


V₀ leads i by a phase difference of $\pi/2$

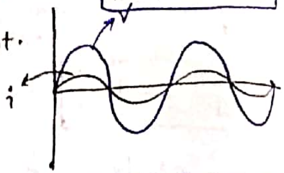
## RMS for 2 currents : $i = i_1\sin\omega t + i_2\cos\omega t$

$$\boxed{i_{rms} = \sqrt{\frac{i_1^2 + i_2^2}{2}}}$$

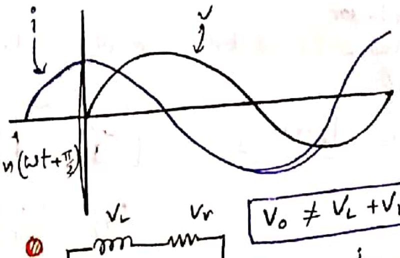
→ Phasor



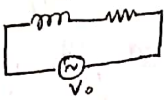
$V = V_0\sin\omega t$



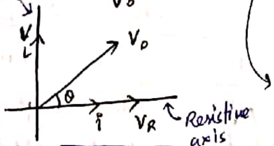
V and i are in phase in purely resistive circuits.



Current leads the voltage by a phase diff of $\frac{\pi}{2}$



$\boxed{V_0 \neq V_L + V_r}$ → cos they are not in same phase.



$\boxed{\tan\theta = \frac{V_L}{V_R} = \frac{X_L}{R}}$

V will lead by θ

$V_0 = \sqrt{V_R^2 + V_L^2}$

$V_0 = V_R\hat{i} + \hat{j}\,V_L$

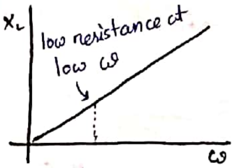
| | |
|---|---|
| $V_R = i_0 R$ ✓ | $V = iR$ |
| $V_L = i_0 X_L$ | $V = iX_L$ |
| $V_C = i_0 X_C$ | $V = iX_C$ ✗ |

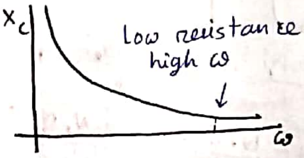
$i_0 Z = \sqrt{(i_0 R)^2 + (i_0 X_L)^2}$

$Z = \sqrt{R^2 + (X_L)^2}$

Impedence → combined resistance offered by the inductor and resistor

- Inductive circuits are used to make low pass filter
- Capacitive circuits are used to make high pass filter



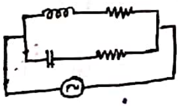


- If $X_L = X_C$ (Resonance) $Z = R$ , $i_{max} = \frac{V_0}{R}$

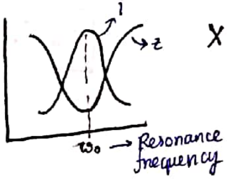
$\omega L = \frac{1}{\omega C} \rightarrow \omega = \frac{1}{\sqrt{LC}}$ Resonance frequency → $f = \frac{1}{2\pi\sqrt{LC}}$

$V_L = iX_L$ ; $V_C = iX_C$ [They have phase difference π]

- At resonance, the PD across the inductor nullifies the potential drop taking place across the capacitor
- Series LCR circuits are also known as Acceptor circuits



$\frac{1}{Z} = \frac{1}{R + jX_L} + \frac{1}{R - jX_C}$ ; $Y = \frac{R - jX_L}{R^2 + X_L^2} + \frac{R + jX_C}{R^2 + X_C^2}$

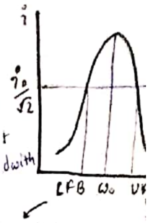


ω₀ → Resonance frequency

$X = \omega L - \frac{1}{\omega C}$

- When $\omega < \omega_0$ and the ω is ↑creasing, then Z is ↓creasing
- When $\omega > \omega_0$ ω is ↓creasing, Z is ↑creasing.

## Quality Fac



$\omega_0 - \Delta\omega$

ower frequency (u band)

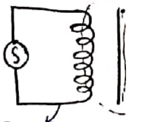
## Power dissep

$i = i_0\sin\omega t$

$\langle P\rangle = \frac{\int_0^T P\,dt}{\int_0^T dt}$

## Transform

- Transform



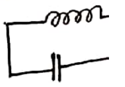
Primary

$e_P = -N_P$

$e_S = N_S$

In ideal case

## LC Oscilla



General equ

$\boxed{q = q_0\sin}$

## Potential e

$U_S = \frac{1}{2}k$

If t = 0, ca

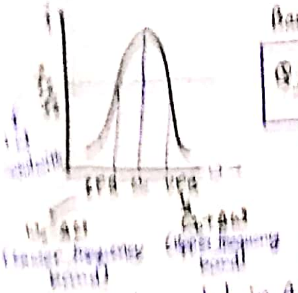
If C is

- $K = \frac{1}{2}mv$

[Mass opposes in state

[Capacitor

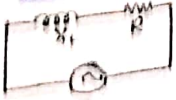
## Quality factor (Q factor):



Bandwidth, BW = UFB − LFB = 2Δω   ∴ Q factor = $\frac{\omega_0}{2\Delta\omega}$

$\boxed{Q\text{ factor} = \frac{\omega_0 L}{R}}$

## Choke Coil:



- It assist in the starting of the tube light without dessepation of power, $\cos\phi = \frac{R}{\sqrt{X_L^2+R^2}} = \frac{R/X_L}{\sqrt{1+\left(\frac{R}{X_L}\right)^2}}$

As $X_L >> R$  ∴ $\cos\phi = 0$

∴ $P = 0$

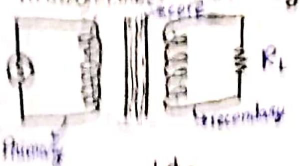
## Power desspated in AC circuits:

$i = i_0 \sin\omega t$ ; $V = V_0 \sin(\omega t + \phi)$ , $P = iV = V_0 i \sin(\omega t + \phi) \sin\omega t$.

$\langle P \rangle = \frac{\int P\,dt}{\int dt} = \frac{V_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos\phi$ ; $\cos\phi = \frac{R}{\sqrt{R^2+X^2}} = R/Z$ → $\boxed{P = V_{rms}\, i_{rms}\, R/Z}$

## Transformer:

- Transformers work by the principle of mutual inductance.



- Step Up: $N_s > N_p$ → $e_s > e_p$ ; $P = ei$
- Step Down $N_p > N_s$ → $e_p > e_s$

∴ $\boxed{\frac{i_s}{i_p} = \frac{N_p}{N_s}}$  $\boxed{i \propto 1/N}$

$e_p = -N_p \frac{d\phi_p}{dt}$

$e_s = N_s \frac{d\phi_s}{dt}$

In ideal case, $\phi_p = \phi_s$

∴ $\boxed{\frac{e_p}{e_s} = \frac{N_p}{N_s}}$  $\boxed{e \propto N}$

Efficiency $\boxed{\eta = \frac{e_s i_s}{e_p i_p} \times 100}$

## LC Oscillation

$-L\frac{di}{dt} - \frac{q}{c} = 0$

$\frac{di}{dt} = -\frac{q}{LC}$

General equation $\boxed{\frac{d^2q}{dt^2} = -\frac{q}{LC}}$

$\boxed{q = q_0 \sin(\omega t + \phi)}$

- Comparison:

| SHM | L.C.O. |
|---|---|
| $x = A\sin(\omega t + \phi)$ | $q = q_0 \sin(\omega t + \phi)$ |
| $v = A\omega\cos(\omega t + \phi)$ | $i = q_0\omega\cos(\omega t + \phi)$ |
| $v = \omega\sqrt{A^2 - x^2}$ | $i = \omega\sqrt{q_0^2 - q^2}$ |
| $a = -\omega^2 A\sin(\omega t + \phi)$ | $\frac{di}{dt} = -\omega^2 q_0 \sin(\omega t + \phi)$ |

## Potential energy:

$U_s = \frac{1}{2}kx^2$ , $U_c = \frac{1}{2}\frac{q^2}{c}$

$\underbrace{\qquad\qquad}_{\text{Analogous}}$

If t = 0, capacitor is charged, take (sin) 

If C is at 0 charge, take cos 

- $K = \frac{1}{2}mv^2$ , $U_L = \frac{1}{2}Li^2$

$\underbrace{\qquad\qquad}_{\text{Analogous}}$

[Mass opposes change in state] [Inductor opposes change in current]

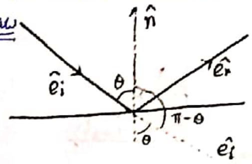
[Capacitor provides driving force]

# OPTICS

## Reflection:

### Laws of reflection in vector form:

1st Law



$\hat{e_i} = \sin\theta\hat{i} + \cos\theta(-\hat{j})$

$\hat{e_r} = \sin\theta\hat{i} + \cos\theta\hat{j}$

$\hat{e_r} - \hat{e_i} = 2\cos\theta\hat{j} = 2\cos\theta\hat{n}$

$\hat{e_i}\cdot\hat{n} = \cos(\pi-\theta)$

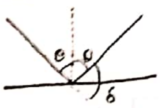
$\cos(\theta) = -\hat{e_i}\cdot\hat{n}$

$\hat{e_r} - \hat{e_i} = -2(\hat{e_i}\cdot\hat{n})\hat{n}$

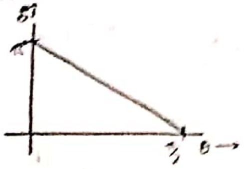
$\boxed{\hat{e_r} = \hat{e_i} - 2(\hat{e_i}\cdot\hat{n})\hat{n}}$

2nd Law: $\boxed{[\hat{e_i}\,\hat{n}\,\hat{e_r}] = 0}$

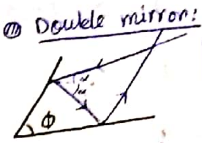
### Deviation produced by plane mirror:



$\delta = \pi - 2\theta$



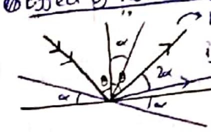
### Double mirror:



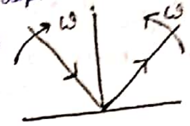
$\boxed{\delta = 2\pi - 2\phi}$

→ Independant of incident angle but dependant on angle b/w the mirror

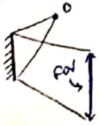
### Effect of Rotation of mirror:



→ Keeping the incident ray fixed, if mirror is rotated by $\alpha$, then the reflected ray rotates by $2\alpha$

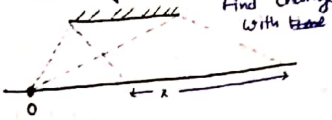


### Field of View (FOV):

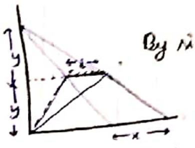


Q) mirror in free fall

Find change in FOV(x) with ~~time~~ y



→

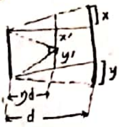


By similarity, $\frac{y}{\ell} = \frac{2y}{x}$

$\therefore \boxed{x = 2\ell}$ → independant of y.

- mirror of length $2\ell$ is needed for a man to see his whole body whose height is $\ell$.

Q) Height of mirror to see an object behind.



$\frac{x'}{x} = \eta = \frac{y'}{y}$ $\therefore x' = \eta x$, $y' = \eta y$

$h_{mir} = \eta(x+y)$

$h_{obj} = (x+y)(\eta+1)$

$\boxed{\therefore H_{mirror} = \left(\frac{\eta}{\eta+1}\right)H_{object}}$

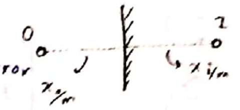
### Img velocity w.r.t to plane mirror.

$x_{o/m} = -x_{i/m} \rightarrow x_o - x_m = -(x_i - x_m)$

$x_i = 2x_m - x_o$

only for $v \perp$ to mirror

$\therefore \boxed{v_i = 2v_m - v_o}$



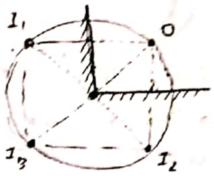
### No of images in case of multiple mirrors:

$n = \frac{360}{\theta}$

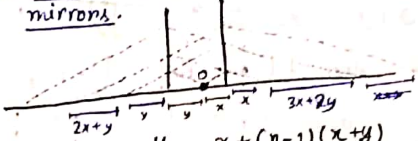
| n | obj placed symmetrically | Asymmetrically |
|---|---|---|
| odd | $N = n-1$ | $N = n$ |
| even | $N = n-1$ | $N = n-1$ |
| fraction | $N = [n]$ | $N = [n]$ |

→ special case ($\theta = 90°$)

O, $I_1, I_2, I_3$ are ~~cyclic~~ concyclic (centre at origin)



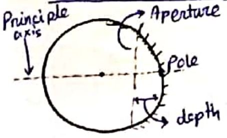
### Consecutive reflections in parallel mirrors.

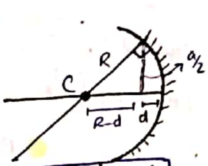


$y_n = x + (n-1)(x+y)$

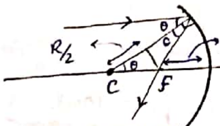
$y_n = y + (n-1)(x+y)$

### Spherical Mirror:



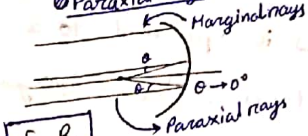


$\boxed{R = \frac{d^2 + \frac{a^2}{4}}{2d}}$



$\boxed{f = \frac{R(1-\sec\theta)}{2}}$ → $\boxed{f = \frac{R}{2}}$

### Paraxial Rays



### Sign Convention:

- All dist are measured from pole
- Dist measured in the direction of incident ray is taken as positive and opposit as negative
- Dist measured above the principle axis is +ve, below is -ve.

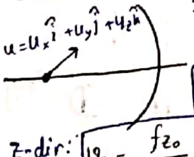
### Mirror Formula:

$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$ → obj dist, img dist.

### Magnefication: ← ~~lateral~~

$\boxed{m = \frac{h_i}{h_o} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}}$

### Image velocity w.r.t. spherical mirror:

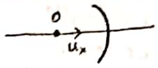


x-dir: $\boxed{v_x = -(m_t)^2 u_x}$

y-dir: $\boxed{v_y = \frac{f y_o}{(f-x_o)^2}u_x + \frac{f}{f-x_o}u_y}$

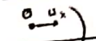
z-dir: $\boxed{v_z = \frac{f z_o}{(f-x_o)^2}u_x + \frac{f}{f-x_o}u_z}$

Case I ($y = 0, v_y = 0$)



$v_x = -m_t^2 u_x$

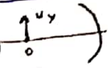
Case II ($y \neq 0, v_y = 0$)



$v_y = \frac{f y_o}{(f-x_o)^2}u_x$

If mirror is moving take rel velocity w.r.t mirror first.

Case III ($y = 0, v_y \neq 0, v_x = 0$)



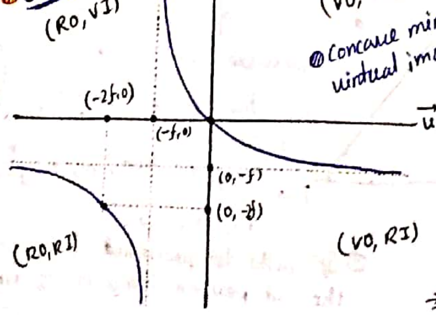
$v_y = \frac{f}{f-x_o}\cdot u_y$

### (Longitudinal Mag) extended object:

$\boxed{\frac{dv}{du} = -\frac{v^2}{u^2} \qquad m_l = -m_t^2}$

- Focal length of mirror never depends on medium

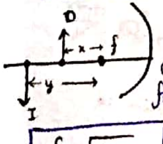
### Concave



- Concave mirror can't form a virtual image of virtual object.

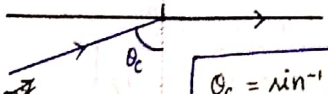
### Newton's formula:

↓ measuring dist from focus



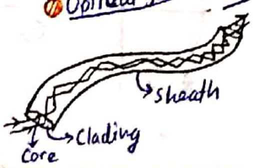
$\boxed{f = \sqrt{xy}}$

### Total Internal Reflection (TIR)

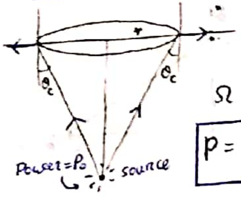


$\boxed{\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)}$

### Optical fibre



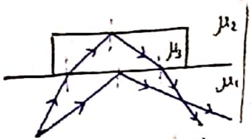
## Circle of Illuminance:



$$r = \frac{h}{\sqrt{\mu^2-1}}$$

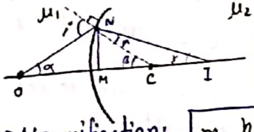
$$\Omega = 2\pi(1-\cos\theta_c)$$

$$\boxed{P = \frac{\Omega}{4\pi}P_0 = \frac{1-\cos\theta_c}{2}P_0}$$



If a ray of light is travelling from $\mu_1$ to $\mu_2$ and TIR takes place then TIR will nessecerily take place irrespective of the $\mu$ of 3rd medium placed b/w $\mu_1$ and $\mu_2$

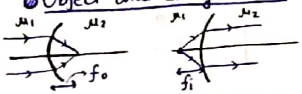
## Refraction on curved surfaces:



$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}}$$

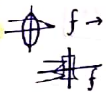
**Magnification:** $\boxed{m = \frac{h_i}{h_o} = \frac{\mu_1}{\mu_2}\cdot\frac{v}{u}}$

**Object and Image Focus:**



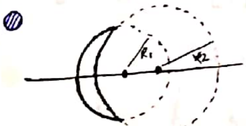
$$\boxed{\frac{\mu_2}{f_o} = \frac{\mu_1}{f_i}}$$

**lens formula:** $\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$



## Lens Makers' formula

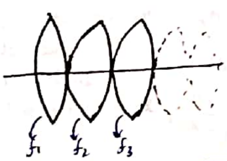
$$\boxed{\frac{1}{f} = \left(\frac{\mu_{Lens}}{\mu_{surr}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$



$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\to \oplus ve \to$ convex lens with one surface concave $\to$ concavoconvex

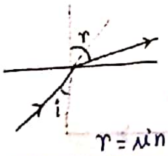
## Combination of lenses



All the focal lengths are to be calculated in the same sorrounding media

$$\boxed{\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \cdots}$$

## Deviation in case of TIR



$\delta = |i - R|$

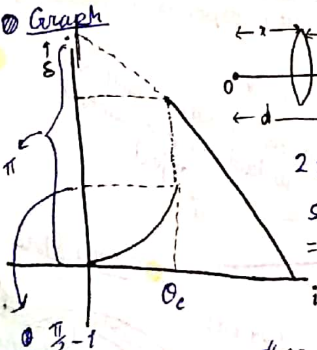
$\delta = i - r$

$\mu_1 \sin i = \mu_2 \sin r$

$r = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\sin i\right)$

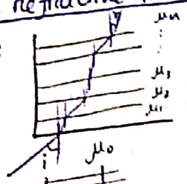
$$\boxed{\delta = i - \sin^{-1}\left(\frac{\mu_1}{\mu_2}\sin i\right)}$$

**Graph**



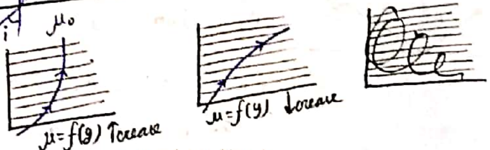
## Variable refractive index

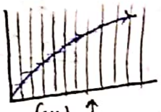
**Discrete:**



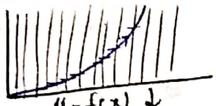
$$\boxed{\mu_0 \sin i = \mu_n \sin r_n}$$

**Continuous:**



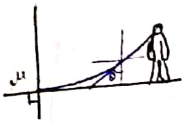


$\mu = f(x)$ ↑



$\mu = f(x)$ ↓

**Q)** How far can the man see the road? $\mu = \sqrt{1+ay}$



$1 \sin\frac{\pi}{2} = \mu \sin\theta$

$\sin\theta = \frac{1}{\mu}$

$\tan(90-\theta) = \frac{dy}{dx}$

$\cot\theta = \frac{dy}{dx}$

$\leftarrow \sqrt{\mu^2-1} = \frac{dy}{dx}$

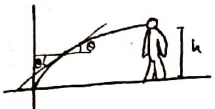
$\sqrt{ay} = \frac{dy}{dx}$

$\int y^{-1/2}\,dy = \sqrt{a}\int dx$

$\Rightarrow 2\sqrt{y} = \sqrt{a}\,x \to \boxed{y = \frac{a}{4}x^2} \to$ Parabola

$\to$ Put $y = 2m$, find x

**Q)** $\mu = \sqrt{1-ay}$



$\tan\theta = \frac{dy}{dx}$ Then proceed accordingly.

**for convex lens (★)**

if $\mu_l > \mu_s$: $f \to \oplus ve$ i.e. lens is converging

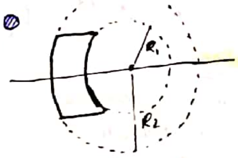
if $\mu_l < \mu_s$: $f \to \ominus ve$ i.e. lens is diverging

$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{-R_2}\right)$

$$\boxed{\frac{1}{f} = (\mu_{rel}-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

D $\to \frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$

[ $\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right)$



$\frac{1}{f} = (\mu_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

$\to \ominus ve$

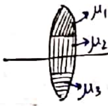
↳ convexo concave

## Power

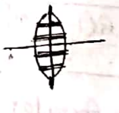
$P = \frac{1}{f(\text{in m})}$

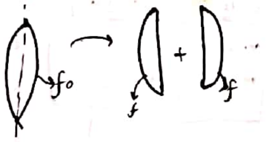
↳ Unit → diopter

**Cutting:** A part of a lens is equivalent to a complex lens.

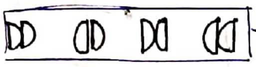


→ forms 3 different images



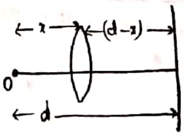


$\frac{1}{f} + \frac{1}{f} = \frac{1}{f_o} \to f = 2f_o$



All arrangements have same focal length

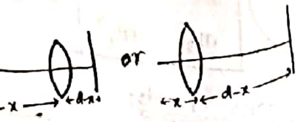
## Displacement methods to find the local length of a lens!



$\frac{1}{d-x} + \frac{1}{x} = \frac{1}{f} \Rightarrow x^2 - dx + df = 0$

$D \geqslant 0$ $\boxed{d \geqslant 4f} \to$ condition

2 possible position →



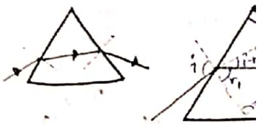
Seperation b/w the 2 positions $= d - 2x$

Magnification, $m_1 = \frac{x}{d-x}$ $m_2 = \frac{d-x}{x}$

$m_1 m_2 = 1$

If in the 1st positioned, image is n times, then in

## PRISM (deviates a



$\delta = i + e - A$

for $\delta_{min}$ $(i=e)$: $\delta_{min} = 2i$

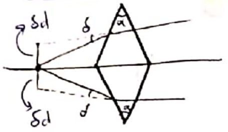
$\Rightarrow i = \frac{A+\delta_m}{2}$ ∵ $\sin i = \mu$

$r_1 + r_2 = A$

$r_1 = r_2$ ∴ $r_1 = \frac{A}{2}$

$\mu = \frac{\sin}{}$

**Fresner's Biprism:**

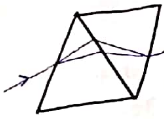


**Double Prism:**

$\theta = A(\mu_v -$

$\delta = A(\mu_y$

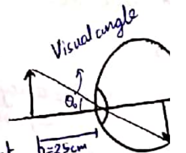
**Achromatic Combina**



$\theta =$

Simil

$\delta =$

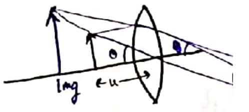


**Eye:**

least dist $D = 25cm$

max $\theta$, $\boxed{\theta_0 = \frac{h_0}{D}}$

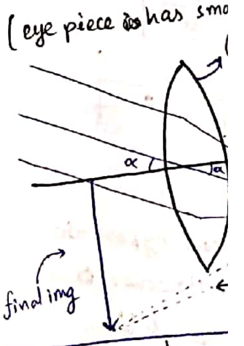
**Simple Microscope:**



$$\boxed{M = \frac{\theta}{\theta_0}}$$

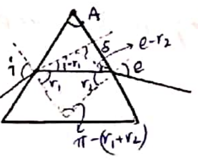
**Telescope:** $\left(M = \frac{\angle}{\angle}\right.$

(eye piece has sma



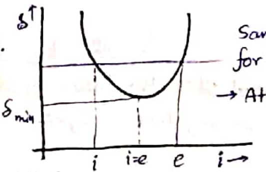
$$M = \frac{\beta}{\alpha} = \frac{h_0/u_e}{h_0/f_o}$$

## PRISM (deviates a ray of light twice towards its base)



$\boxed{\delta = i+e-A}$ Doesn't always hold.

$\boxed{r_1+r_2=A}$



Same deviation occurs for 2 angle of incidence

→ At $\delta_{min}$, (i) $i=e$
(ii) $r_1=r_2$
(iii) After 1st refraction, the ray is ∥ to base

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- $\delta = i+e-A$

For $\delta_{min}$ $(i=e)$: $\delta_{min} = 2i-A$ ☆

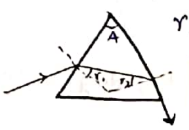
$\Rightarrow i = \frac{A+\delta_m}{2}$ $\because \sin i = \mu \sin r$

$r_1+r_2=A$

$r_1=r_2$ $\therefore r_1 = \frac{A}{2}$

$$\boxed{\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin \frac{A}{2}}}$$ ★

### Condition for no Emergance:



$r_{2_{min}} = \theta_C$

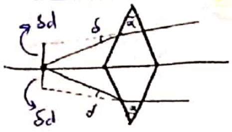
$$\boxed{\mu = \operatorname{cosec}\left(\frac{A}{2}\right)}$$

### Small Angle Prism:

$A < 6°$ $\delta = i+e-A$

$$\boxed{\delta = A(\mu-1)}$$

### Fresner's Biprism:



### Dispertion:

$\boxed{\mu \propto \text{deviation}}$

### Cauchy's Equation,

$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$

$\lambda_{red} > \lambda$ violet

$\mu_{red} < \mu_{violet}$

$\delta_r = A(\mu_r-1)$

$\delta_v = A(\mu_v-1)$

Angular dispertion

$$\boxed{\theta = \delta_v - \delta_r = A(\mu_v-\mu_r)}$$

Yellow is the average (w.r.t λ)

$\delta_y = A(\mu_y-1)$

$\boxed{\frac{\theta}{\delta_y} = \omega}$ → Dispersive power

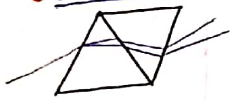
$$\boxed{\therefore \omega = \frac{\mu_v-\mu_r}{\mu_y-1}}$$

### Double Prism:

$$\boxed{\theta = A(\mu_v-\mu_r) - A'(\mu_v'-\mu_r')}$$

$$\boxed{\delta = A(\mu_y-1) - A'(\mu_y'-1)}$$

### Direct vision Prism (Dispersion w/o deviation)

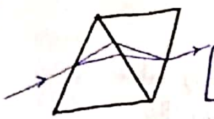


$\delta = 0$, $A(\mu_y-1) = A'(\mu_y'-1) \rightarrow A' = \frac{A(\mu_y-1)}{(\mu_y'-1)}$

$\theta = A(\mu_v-\mu_r) - \frac{A(\mu_y-1)}{\cancel{(\mu_y'-1)}}\cancel{(\mu_y'-1)}$

$$\Rightarrow \boxed{\theta = A(\mu_y-1)(\omega-\omega')}$$

### Achromatic Combination:



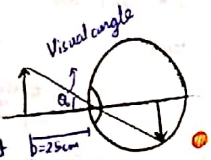
$\theta = 0$

Similar process.

$$\boxed{\delta = A(\mu_y-1)\left[1-\frac{\omega}{\omega'}\right]}$$

## ⋘ Optical Instruments ⋙

### Eye:



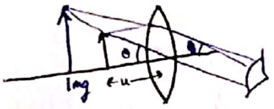
least dist D=25cm

max θ, $\boxed{\theta_0 = \frac{h_0}{D}}$

### Microscope:

Magnifying Power, $\boxed{M = \frac{\angle \text{ subtended by img at eye}}{\angle \text{ subtended by object placed at D}} = \frac{\theta}{\theta_0}}$

### Simple Microscope:



$$\boxed{M = \frac{\theta}{\theta_0}}$$

### Relaxed Eye (Normal adjustment):

$u=f$, $\theta = \frac{h_0}{u} = \frac{h_0}{f}$

$$\boxed{M = \frac{h_0/f}{h_0/D} = \frac{D}{f_0}}$$

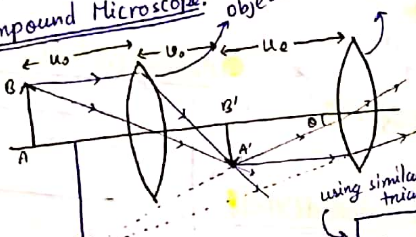
### Strained eye (img at D):

$-\frac{1}{D} - \frac{1}{-u} = \frac{1}{f}$, $\theta = \frac{h_0}{u}$

$\theta = h_0\left(\frac{1}{f}+\frac{1}{D}\right)$

$$\boxed{M = 1+\frac{D}{f}}$$

### Compound Microscope:



$\theta = \frac{A'B'}{u_e}$

$M = \frac{\theta}{\theta_0} = \frac{A'B'/u_e}{AB/D}$

using similar triangles $= \frac{A'B'}{AB} \cdot \frac{D}{u_e}$

$$\boxed{M = \left|\frac{v_0}{u_0}\right| \cdot \frac{D}{u_e}}$$

Normal adjustment: $(u_e = f_e)$

$$\boxed{M = \left|\frac{v_0}{u_0}\right| \frac{D}{f_0}}$$

Strained Adjustment: $\left(\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}\right)$

$$\boxed{M = \left|\frac{v_0}{u_0}\right|\left(1+\frac{D}{f_e}\right)}$$

As $\left|\frac{v_0}{u_0}\right| > 1$

$\therefore$ $\boxed{M_{compound} > M_{simple}}$

### Length of microscope: $L = v_0 + u_e$

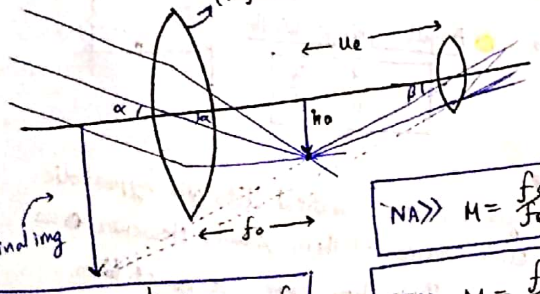
$$\boxed{\begin{array}{ll} NA \gg & L = v_0 + f_e \\ SE \gg & L = v_0 + \frac{D f_e}{D+f_e} \end{array}}$$

### Telescope:

$\left(M = \frac{\angle \text{ sub by img}}{\angle \text{ sub by obj viewed directly}}\right)$

(eye piece has small aperture, f)

(objective has large aperture, f)



$$\boxed{M = \frac{\beta}{\alpha} = \frac{h_0/u_e}{h_0/f_0} = \frac{f_0}{u_e}}$$

$$\boxed{NA \gg \; M = \frac{f_0}{f_e}}$$

$$\boxed{SE \gg \; M = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)}$$

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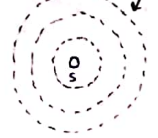
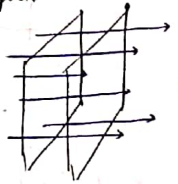
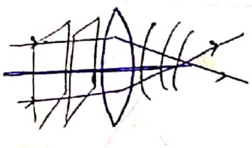
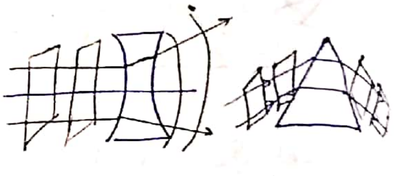
# Wave Optics

**Wave front:** ~~Wave front~~ It is the locus of all the points that are vibrating in the same phase. All at 90° with propagation of light.









**Huygen's Principle:**

→ Every point on primary wave front acts a a fresh source of light emitting disturbances known as secondary wavelets

→ These wavelets travel with the speed of light in air

→ The new position of the wave front is given by the geometrical envelope to these secondary wavelets.

∅ Laws of reflection and refraction can be proved with ~~to~~ Huygens Principle.

**Principle of Superposition**

$y_1 = A_1 \sin \omega t$

$y_2 = A_2 \sin(\omega t + \phi)$

$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}}$

$y_{net} = y_1 + y_2$

∅ For $A_{max}$: $\phi = 0, 2\pi, 4\pi, \dots$ $(2n\pi)$

Constructive interference $\hookrightarrow$ $\boxed{\begin{array}{l} A_{max} = A_1 + A_2 \\ I \propto A^2 \\ I_{max} = (A_1 + A_2)^2 \end{array}}$

∅ For $A_{min}$: $\phi = \pi, 3\pi, \dots$ $(2n+1)\pi$

$A_{min} = |A_1 - A_2|$

$\boxed{I_{min} = (A_1 - A_2)^2}$

∅ $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

$(A^2 \propto I) \rightarrow I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

If $I_1 = I_2$,

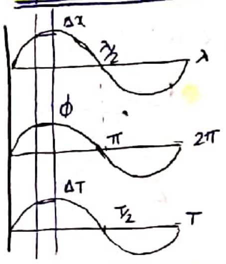
$I = 2I_0 + 2I_0\cos\phi$

$= 4I_0\cos^2\phi/2$

$\frac{\phi}{2\pi} = \frac{\Delta x}{\lambda} \rightarrow \phi = \frac{2\pi\Delta x}{\lambda}$

★ ∴ $\boxed{I = 4I_0\cos^2\left(\frac{\pi}{\lambda}\cdot\Delta x\right)}$ ★

**Representing Against Different Variables:**



★ ∴ $\boxed{\frac{\Delta x}{\lambda} = \frac{\phi}{2\pi} = \frac{\Delta T}{T}}$ ★

For CS

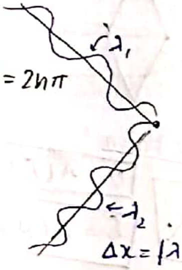
$\frac{\Delta x}{\lambda} = \frac{\phi}{2\pi} \curvearrowright \phi = 2n\pi$

$\Delta x = n\lambda$

For DS

$\frac{\Delta x}{\lambda} = \frac{(2n-1)\pi}{2\pi}$

$\Delta x = \frac{(2n-1)}{2}\lambda$



$\Delta x = |\lambda_1 - \lambda_2|$

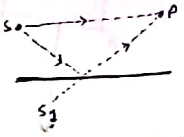
**Interference:** It is the re-distribution of light energy when light from 2 coherent sources superimpose with each other.

∅ **Conherent sources,** ⟶ + const or no ϕ ] ⟶ mandatory
+ same f
+ same or almost same A → if not, coherency will be poor.

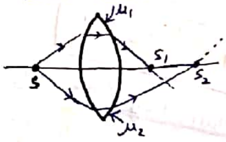
**Making Coherent sources:**

∅ YDSE

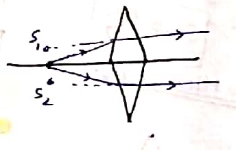
∅ Lloyd's Mirror



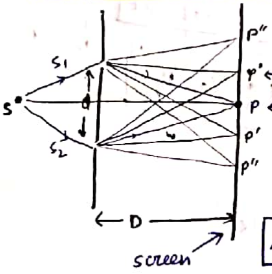
∅ Horizontal Slits



∅ Fresnel's biprism



**Young's Double Slit Experiment (YDSE)**

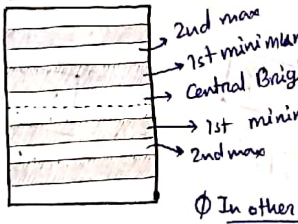


minima

maxima (path diff = 0 $S_1P = S_2P$)

$[D \gg d \gg \lambda]$

$\boxed{\Delta x = d\sin\theta = S_2P - S_1P}$

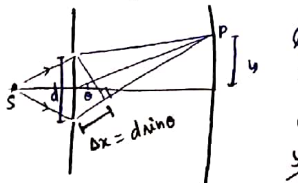


**Fringe Width:**

$\boxed{\beta = y_n - y_{n-1} = \frac{\lambda D}{d}}$, $\boxed{\begin{array}{l} \beta \propto D \\ \beta \propto \frac{1}{d} \end{array}}$

∅ In other medium:—

$\beta_{med} = \frac{\lambda}{\mu}\cdot\frac{D}{d}$ ∴ $\boxed{\beta_{med} = \frac{\beta_{air}}{\mu}}$



∅ For CI:

$d\sin\theta = n\lambda$

$d\tan\theta = n\lambda$

$\frac{y}{D} = \frac{n\lambda}{d}$

$\boxed{y = \frac{n\lambda D}{d}}$

Coordinate of nth maxima

∅ If white light is used, CBF will be white, adjacent will be violet, furthest will be red, all colours may not be present.

**Angular fringe Width:**

$\boxed{\theta = \frac{\beta}{D} = \frac{\lambda}{d}}$

Angle θ is generalised bcz to scale it really looks like

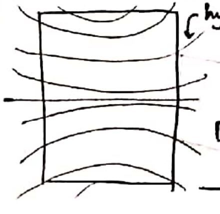
∅ For DI:

$d\tan\theta = (2n-1)\frac{\lambda}{2}$

$\boxed{y = \frac{(2n-1)\lambda D}{d}}$

Position of nth minima

**Geometry of fringes:**

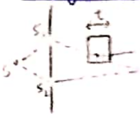


hyperbolic

→ The fringes in case of vertical slits are hyperbolic in shape, with source at their focus. However as D is very large, thus they appear to be st. lines.

→ In case of horizontal slits, the fringes are semicircular is shape.

Fring Shi

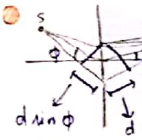


$T_1 = \frac{t}{c}$

$T_2 = \frac{t}{c/\mu}$

$\Delta T = T_2 - T_1$

$\Delta x = c\Delta t =$



∴ $\Delta x = d$

Diffraction:

It is the phe
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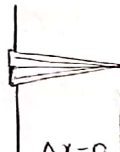
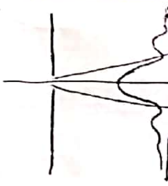
halfes. So
the path



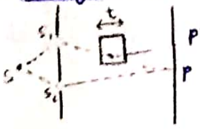
$\Delta x = 0$

adds up

Width of C



**Fring Shift:** $\Delta x = S_2P - S_1P'$ , $S_1P' = S_1P + t(\mu-1)$



$\Delta x = S_2P - S_1P - t(\mu-1)$

$\boxed{\Delta x = d\sin\theta - t(\mu-1)}$

**No of fringes shifted:** $\boxed{N = \frac{FS}{FW} = \frac{t}{\lambda}(\mu-1)}$ → Inversely with λ

→ Slab inserted in upper slit, the fringes shift upward,
→ slab inserted in lowerslit, ―――― downward,

$T_1 = \frac{t}{c}$

$T_2 = \frac{t}{c/\mu} = \frac{\mu t}{c}$

$\Delta T = T_2 - T_1 = \frac{t}{c}(\mu-1)$

$\boxed{\Delta x = c\Delta t = t(\mu-1)}$

ϕ for CI:

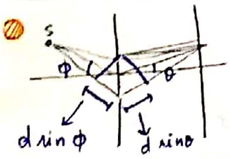
$n\lambda = d\sin\theta - t(\mu-1)$ → If in both, $S_1P' = S_1P + t_1(\mu_1-1)$ ; $S_2P'' = S_2P + t_2(\mu_2-1)$

$d\sin\theta = n\lambda + t(\mu-1)$ → $\Delta x = d\sin\theta + \boxed{t_2(\mu_2-1) - t_1(\mu_2-1)}$ → +ve ↓ → −ve ↑

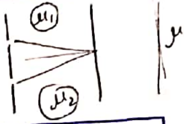
$\tan\theta = \frac{n\lambda}{D} + \frac{t(\mu-1)}{D}$

$\boxed{y_n' = \frac{n\lambda D}{d} + \frac{tD}{d}(\mu-1)}$

$\boxed{FS = y_n' - y_n = \frac{tD}{d}(\mu-1)}$

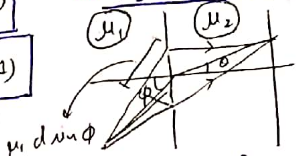


$\therefore \boxed{\Delta x = d\sin\phi + d\sin\theta}$



$\boxed{\Delta x = \mu_1 S_1P - \mu_2 S_2P}$

**Different μ:**



$\boxed{\Delta x = \mu_1 d\sin\phi + \mu_2(-d\sin\theta)}$

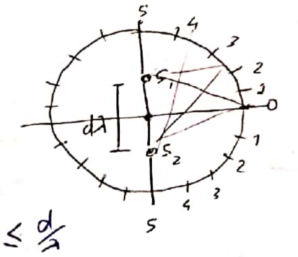
**No of fringes**

$d\sin\theta = n\lambda$

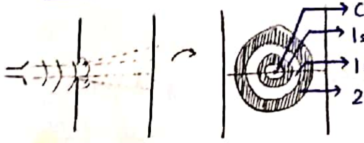
$\sin\theta = \frac{n\lambda}{d}$

$\therefore \frac{n\lambda}{d} \le 1 \rightarrow n \le \frac{d}{\lambda}$

$n < \frac{5\lambda}{\lambda}$ $\therefore n \le 5$

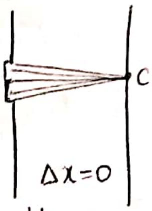


**Diffraction:**

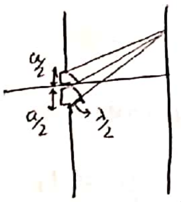


It is the phenomenon of spreading of light into the geometrical shadow region when light encounters a slit whose size is comparable λ

For the formation of 1st secondary minima, imagine the slit to be divided into 2 equal halfes. So that every point in the upper half has a corresponding point in the lower half so the path difference is $\frac{\lambda}{2}$

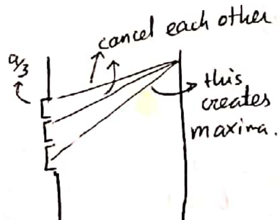


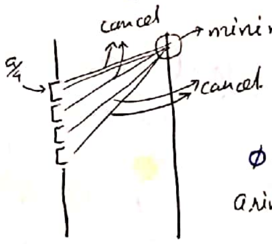
adds up



$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$

$a\sin\theta = \lambda$





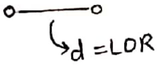
ϕ **For Min:**

$a\sin\theta = \lambda, 2\lambda, ...., n\lambda$

ϕ **For Max:**

$a\sin\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ----, \frac{(2n+1)\lambda}{2}$

**Limit of Resolution:** It is the minimum distance between two objects which can be ~~ɑ~~ distinctly seen with a microscope.

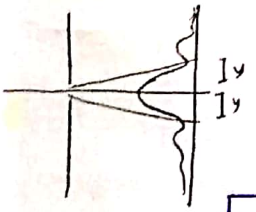


$\boxed{\text{Resoluting power; } RP = \frac{1}{d}}$

$\boxed{\text{Microscope, } RP = \frac{1.22\lambda}{d}}$

$\boxed{\text{Telescope, } RP = \frac{2\mu\sin\theta}{\lambda}}$

**Width of Central Maxima:**

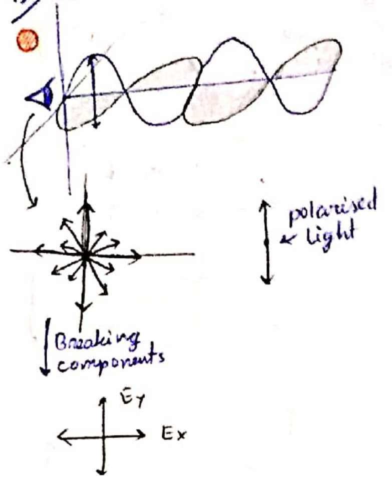


$\tan\theta = \frac{y}{D} = \sin\theta$

$a\sin\theta = \lambda$

$\therefore y = \frac{\lambda D}{a}$

$\therefore \boxed{\text{Width} = 2y = \frac{2\lambda D}{a}}$



## Polarisation

- $E = E_0 \sin(kx - \omega t)$ , $B = B_0 \sin(kx - \omega t)$

$$C = \omega/k = \frac{E_0}{B_0} \quad , \quad \left[ \begin{array}{l} U_E = \frac{1}{2}\epsilon_0 E_0^2 \\ U_B = \frac{B_0^2}{2\mu_0} \end{array} \right] \rightarrow \text{equal}$$

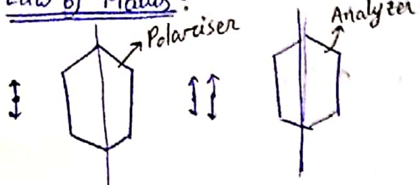
$$\frac{1}{2}\epsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

$$\therefore C = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

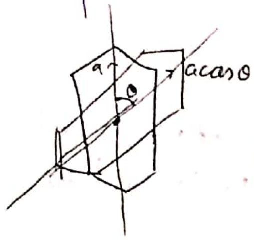
- Electric field vector is responsible for polarisation of light.

- **Polarisation:** It is the phenomenon of restricting the plane of vibration of light to a single plane by passing it thru an optically active crystal.

- **Law of Malus:**



- The intensity of light coming out of the analyzer is directly proportional to the square of the cosine of the angle between the analyzer and polarizer.

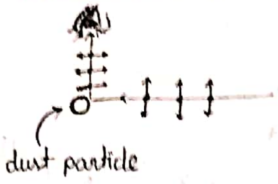


$I_0 = ka^2$

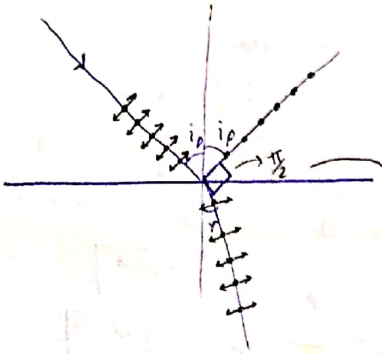
$I = ka^2\cos^2\theta$

$$\boxed{I = I_0 \cos^2\theta}$$

- **Polarisation by scattering** (In nature)



- **Polarisation by reflection** (Brewster's Law)



Generally, all are unpolarised. But, At particular angle, the reflected ray becomes polarised (special condition)

$i_p + r = \frac{\pi}{2}$

$r = \frac{\pi}{2} - i_p$

$\sin i_p = \mu \sin r = \mu \sin(\frac{\pi}{2} - i_p)$

$\sin i_p = \mu \cos i_p$

$$\therefore \boxed{\mu = \tan i_p}$$

↑ Brewster's Law

Ener
Mome
Photo
When
Graph
V vs
%e
If
Stop
rema
Int
Intensity
$I = \frac{P}{A}$
E = Nh
∴ I =
Radiati
E = Nh
$E_t$ =

# Dual Nature of Light

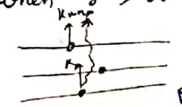
- Energy of photon ~~=~~, $E = h\nu$
- Momentum of photon, $p = h/\lambda$

- **Photons:**
  - Rest mass $= 0$
  - Neutral particle
  - Works on the principle of "all or none"
  - Work function $(\phi) = h\nu_0 = \frac{hc}{\lambda_0}$
    - $\nu_0$ → Thershhold freq [min freq for PE effect]
    - $\lambda_0$ → Threshold wavelength [max wavelength]

## Photoelectric effect:

When $\nu > \nu_0$ → electrons are ejected with $K_{max} = h\nu - h\nu_0$

known as photoelectron



$$\boxed{0 \leqslant K \leqslant h(\nu - \nu_0)}$$

cos they collide w/ other $e^-$

## Stopping Potential:

It is the -ve potential that has to be applied across a electrode so that even the fastest moving electron is unable reach the other plate.

$eV = K_{max}$ $\therefore$ $\boxed{eV = h\nu - h\nu_0}$ → Einstein's Photo electric Equation.

$e\cancel{V} = \cancel{h\nu} - \phi$

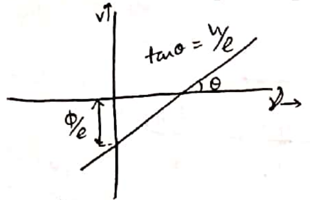
$\cancel{V} = \cancel{\frac{h}{e}}$

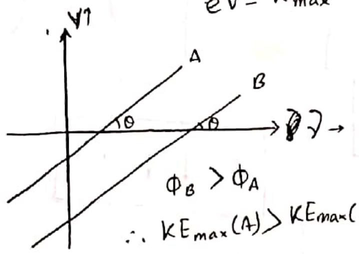
## Graphs:

- V vs $\nu$:

$eV = h\nu - \phi$

$V = \frac{h}{e}\nu - \frac{\phi}{e}$





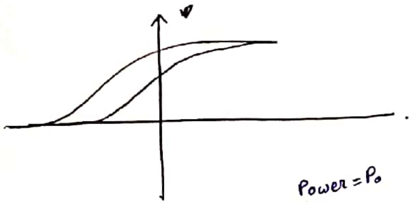
$\phi_B > \phi_A$

$\therefore KE_{max}(A) > KE_{max}(B)$

- If the intensity of light is increased, then the photocurrent increases but the stopping potential remains same.

- If the frequency of light is increased, the Stopping potential increases but the photocurrent remains same.

- $\nu\uparrow \longrightarrow E_p\uparrow \longrightarrow E_{PE}\uparrow \longrightarrow K\uparrow \longrightarrow V\uparrow$



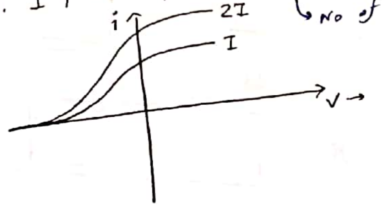
- $I \propto Nh\nu$

N → number of p.c.

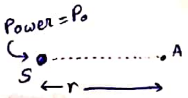
$\therefore I\uparrow \longrightarrow N_p\uparrow \longrightarrow N_{pe}\uparrow \longrightarrow i\uparrow$

$N_p$ → No. of photon

$N_{pe}$ → No of photo $e^-$



## Intensity at a distance:



Intensity at A, $\boxed{I = \frac{P_0}{4\pi r^2}}$

$I = \frac{P}{A} = \frac{E}{At}$

$E = Nh\nu$

$\therefore \boxed{I = \frac{Nh\nu}{At}}$

- ~~Photoelectron~~
- Photon flux density:

$$\boxed{\phi = \frac{N}{At} = \frac{I}{h\nu}}$$

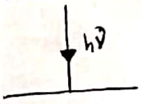
$$\boxed{\frac{N}{t} = \frac{IA}{h\nu}}$$

# If photons have efficiency of $\eta$ $(\eta \leq 1)$

$\frac{\text{No of PE}}{t} = \frac{\eta IA}{h\nu}$

$i = \frac{\text{No of PE}}{t} \times e$ $\therefore$ $\boxed{i = \frac{\eta IAe}{h\nu}}$

## Radiation Pressure:



→ If it's absorbed, $P_i = -h/\lambda$, $P_f = 0$

$\Delta P = h/\lambda$,

$E = Nh\nu$

$\frac{E}{t} = \frac{N}{t}h\nu$

$P = IA = \frac{N}{t}h\nu$

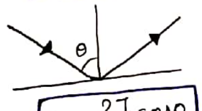
$\therefore \boxed{\frac{N}{t} = \frac{IA}{h\nu}}$

$F = \frac{\Delta P}{\Delta t} = \frac{N}{t} \cdot \frac{h}{\lambda}$

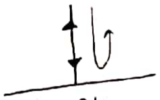
$= \frac{IA}{h\nu} \cdot \frac{h}{\lambda}$

$\therefore \boxed{P = F/A = I/C}$

→ If reflection occurs;



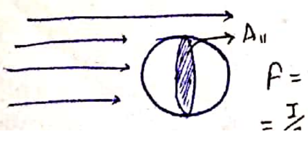
$$\boxed{P = \frac{2I}{C}\cos\theta}$$



$\Delta p = 2h/\lambda$

$$\boxed{P = \frac{2I}{C}}$$

-



$F = PA_{\parallel}$

$= I/c \times \pi R^2$

r,b

**De-broglie Wavelength (Matter waves):**

$K = \frac{p^2}{2m} \rightarrow p = \sqrt{2mK} \rightarrow \frac{h}{\lambda} = \sqrt{2mK}$

$p = h/\lambda$

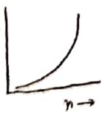
$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$  $\lambda_e = 12.27Å$

**For gas molecules:** $K = \frac{3}{2}kT$

$\boxed{\therefore \lambda = \frac{h}{\sqrt{mkT}}}$

**Radius:** $\boxed{r = \frac{h^2\epsilon_0}{\pi e^2 m} \cdot \frac{n^2}{Z}}$, $\boxed{r \propto \frac{n^2}{Z}}$

$\boxed{r = 0.529 \times n^2/Z}$



$\boxed{T^2 \alpha r^3}$  $\boxed{T \alpha n^3}$

**Kinetic energy:**

$K = \frac{1}{2}mv^2$, $K = \frac{m z^2 e^4}{8n^2h^2\epsilon_0^2}$  $\boxed{K = \frac{me^4}{8h^2\epsilon_0^2}\left(\frac{Z}{n}\right)^2}$

$\boxed{K = |E| = \frac{|U|}{2}}$

**Potential energy:**

$\boxed{U = \frac{-me^4}{4h^2\epsilon_0^2}\left(\frac{Z}{n}\right)^2}$

**Total Energy**

$\boxed{E = \frac{-me^4}{8h^2\epsilon_0^2}\left(\frac{Z}{n}\right)^2}$

$\boxed{E = -13.6 \cdot \frac{Z^2}{n^2} eV}$

**Rydberg's Equation:**

$\boxed{\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$

$\boxed{\frac{1}{\lambda} = \frac{13.6}{hc}Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$

$\boxed{\bar{\nu} = \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$

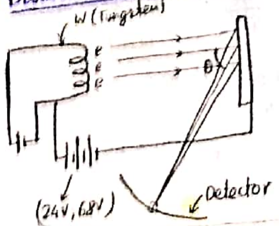
$\boxed{Rhc = 13.6}$

- Lymann
- Balmer
- Paschen
- Brackett
- Pfund
- Humphrey.

**No of spectral lines,** $= \boxed{\frac{\Delta n(\Delta n - 1)}{2}}$

**Davisson & Germer Experiment:** [Proved dual nature of e⁻]

At $\theta = 50°$ & $V = 52V$

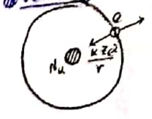


**Atoms:** $e^-$ can stay in those orbits where waves complete.

$2\pi r = n\lambda = \frac{nh}{p}$

$\boxed{mvr = n\frac{h}{2\pi}}$

**velocity:**



$\frac{KZe^2}{r^2} = \frac{mv^2}{r}$

$\boxed{v = \frac{Ze^2}{2\epsilon_0 nh}}$

$= 2.26 \times 10^6 \times \left(\frac{Z}{n}\right)$

**For Hydrogen Atom:**

$E = -\frac{13.6}{n^2} eV$  $E_K = -13.6eV$

$E_L = -3.4eV$

⋮

$E_\infty = 0$

It is the sponten

nuclei with the

**Rate of Disintegr**

$-\frac{dN}{dt} \propto N \rightarrow -\frac{dN}{dt}$ =

decay

$\int_{N_0}^{N} \frac{dN}{N} = -\lambda\int_0^t dt$

$\boxed{N = N_0 e^{-\lambda t}}$ → amount of nuclei remain after time t.

**Half Life:** $N = \frac{N}{}$

$\boxed{t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.}{}}$

**Activity:** $\boxed{R = \lambda N}$

$R = \lambda N_0 e^{-\lambda t}$

$\boxed{R = R_0 e^{-\lambda t}}$

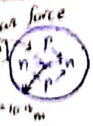
- → Bequerel (Bq) - SI
- → Curie (Cu) - CGS
- → Rd

$\boxed{1\,Bq = \frac{1 \text{ disintegrati}}{\text{sec}}}$

**γ decay:** (Elect

Associated with

# Nucleous



$\boxed{F_n = \frac{Ce^{-kr}}{r^2}}$

**Stability of the nucleous by** Yukawa Theory (Dog-bone Theory)

$p \rightarrow n + \pi^+$ → meson

$n \rightarrow p + \pi^-$

$p \rightarrow p + \pi^0$

**Radius of Nucleous:**

$\boxed{R = R_0 A^{1/3}}$ → $R_0 = 1.15 \times 10^{-15}m$

$V = \frac{4}{3}\pi R_0^3 A$

$\boxed{\rho = \frac{A}{V} \approx 10^{17} kg/m^3}$

independent of Atom

$\boxed{1amu = 1.67 \times 10^{-27} kg}$

**Mass of nucleous:**

→ $(A-Z)M_n + ZM_p$ = Theoritical mass →

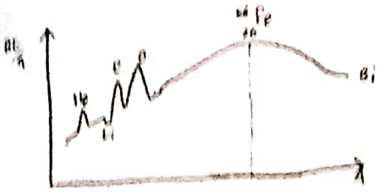
→ $M_{atom} - Zm_e$ = Practicle mass →

$\Delta m = [(A-Z)M_n + ZM_p] - [M_{atom} - Zm_e]$

$E = \Delta mc^2$

↳ Binding energy [Energy released in formatio of nucleous]

$\left(\frac{\text{Binding E}}{\text{At. mass}}\right) \alpha$ stability



- $^{56}_{26}Fe$ nuclei is most stable
- Towards higher atomic numbers, the binding energy is less.
- To increase the binding energy, so the nuclei disintegrate into 2 or more segments,
- So in order to increase the binding energy. process is known as nuclear fission.
- Towards lower atomic numbers, we again see the binding energy is less, so in order to increase the nuclei fuse together to form heavy nuclei and this process is known as nuclear fusion.

**Energy Equivalent:**

$1 amu = 1.6 \times 10^{-27} c^2$

~~1amu~~ $1 amu = 931 MeV$

$E = \Delta m_{(amu)} \times 931 MeV$

# RadioActivitY

- It is the sponteneous disintegration of a heavy nuclei into 2 or more daughter nuclei with the release of enormous amount of energy.

- **Rate of Disintegration:**

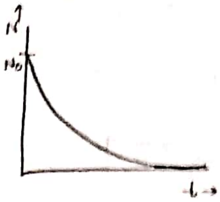
$$\left(-\frac{dN}{dt}\right) \propto N \rightarrow -\frac{dN}{dt} = \lambda N$$

decay const

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

amount of nuclei remaining after time t.



- Probability of surviving $= \frac{N}{N_0} = e^{-\lambda t}$
- Probability of decaying $= 1 - e^{-\lambda t}$

- **Amount decayed:** $N_{decayed} = N_0 - N = N_0(1 - e^{-\lambda t})$

- **Half Life:** $N = \frac{N_0}{2}$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

- **Average life:**

$$\boxed{t_{avg} = \frac{\int t\,dN}{\int dN} = \frac{1}{\lambda}}$$

- **Activity:** $\boxed{R = \lambda N}$

$R = \lambda N_0 e^{-\lambda t}$

$$\boxed{R = R_0 e^{-\lambda t}}$$

- Bequerel (Bq) - SI
- Curie (Cu) - CGS
- Rd

$$\boxed{1\ Bq = \frac{1\ \text{disintegration}}{\text{sec}}}$$

- **α Decay:** ${}^{A}_{Z}X \longrightarrow {}^{Z-4}_{Z-2}Y + \alpha$

$\left({}^{4}_{2}He^{2+}\right)$

- **β⁻ decay ($e^-$):**

$n \longrightarrow p + e^- + \bar{\nu}$ Anti neutrino.

${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + \beta + \bar{\nu}$

- **β⁺ decay:**

$p \longrightarrow n + e^+ + \nu$ → neutrino

${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu$

- **Electron Capture:** (K capture)

${}^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}Y$

- **Q Value:** $a + X \longrightarrow Y + b$ → By product.

Projectile, Target, Product

$$Q_{value} = \left[(m_a + m_X) - (m_Y + m_b)\right]c^2$$

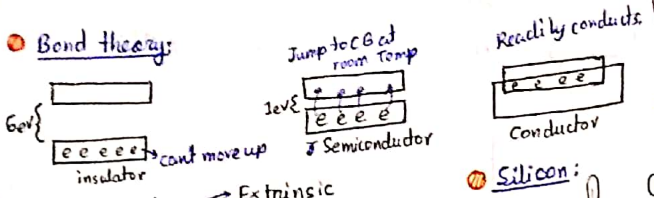
- **γ decay:** (Electromagnetic wave)

Associated with both α and β decay.

# Semiconductors

- Electrical devices → works on high voltage
- Electronic devices → replaced by processor/IC/RAM

## Bond theory:

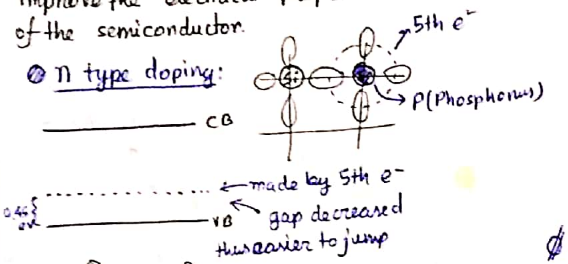


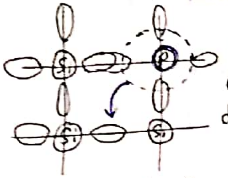
- Semiconductor → Extrinsic / Intrinsic (Pure form)

## Doping:

It is the process of intentional mixing of impurities to improve the electrical properties of the semiconductor.

### n type doping:



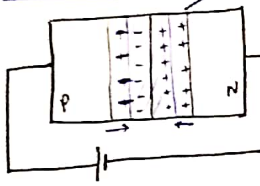


When the 5th e⁻ moves to a diff position.

P is ⊕ve and attracts a e⁻ out of Si. Thus a hole is created.

$\boxed{n_e > n_h}$ → n type (doping with Gr. 15 elements)

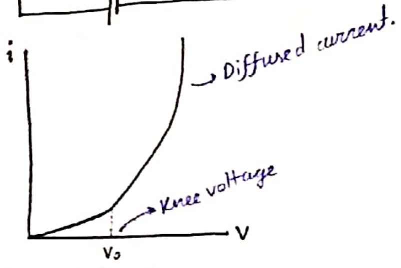
## Forward Biasing:



The depletion region shrinks

Thus the resistance decreases and the current increases.

(Resistance = 0)



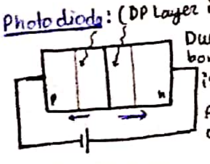
## Types of Diodes:

- **Light Emitting Diode:** LED is always used under forward biased condition

(e-h) bonds are broken due to energy provided by the battery. This cleavage releases light

Ga, As, In, Phosphide

- **Photodiode:** (DP Layer increases)

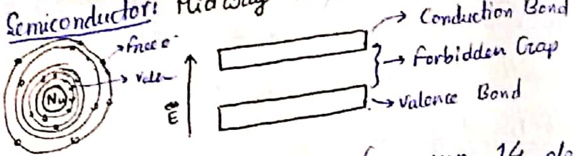


Due to breaking of e-h bond a mild current is produced.

Always used under reverse biased condition.

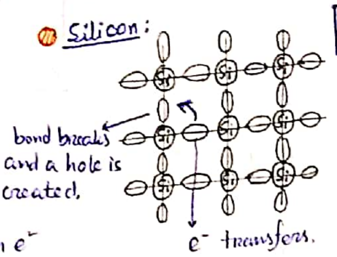
- Solar Cell is Collector of large no of photo diode.

## Semiconductor: Midway between insulator and conduct.



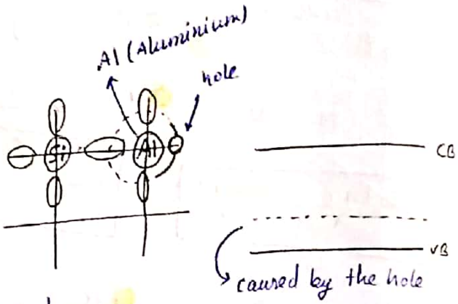
- Semiconductors are made of group 14 elements

## Silicon:



$\boxed{n_e = n_h}$

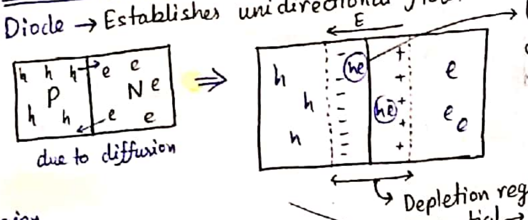
### p type doping:



- Both types of Semiconductors are electrically neutral:
  - n type → cos -ve carriers ($e^-$) are majority.
  - p type → cos +ve carriers (holes) are majority.

## P-N Junction (Junction Diode):

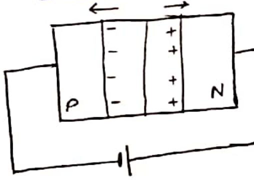
Diode → Establishes unidirectional flow.



(h-e) are paired, charges exist but no charge carriers.

Depletion region [Potential → Barrier Potential.

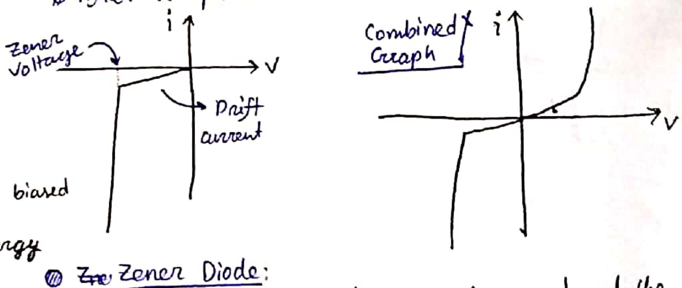
### Reverse Biasing:



[Resistance = ∞]

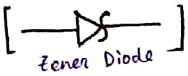
Some of the minority carriers sneak through causing a negligible current called drift current.

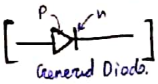
- After a particular V, the field causes breakdown



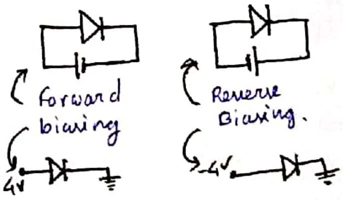
### ~~Zre~~ Zener Diode:

- It is a specially designed diode to operate at the zener voltage.
- It is heavily doped.

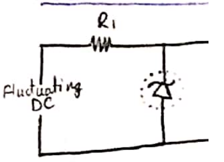

[Zener Diode]



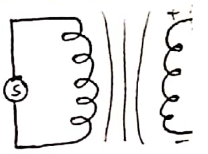
[General Diode.]



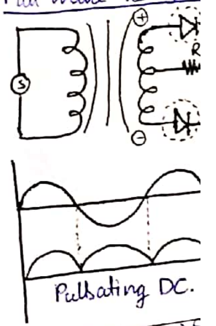
## Zener Diode

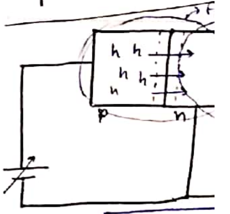


## Diode as a rec



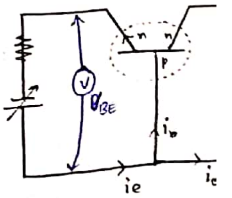
## Full Wave Rectifie



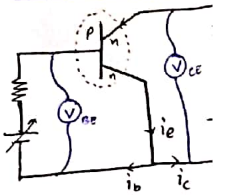


$\boxed{i_e = i_b +}$

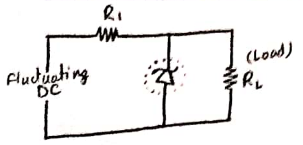
## Common Base Mod

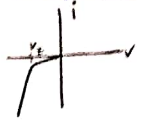


## Common Emitter M



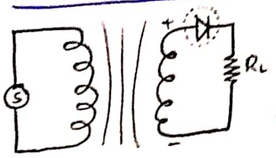
## Zener Diode As voltage Regulator:



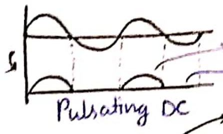


- When voltage exceeds working voltage of $R_L$, the breakdown occurs and i becomes very large. Major drop occurs at $R_1$ and decreases the voltage. Then the zener diode becomes functional again
- Initially the Z.D provides infite resistance thus no current

## Diode as a rectifier (Half Wave re difier):



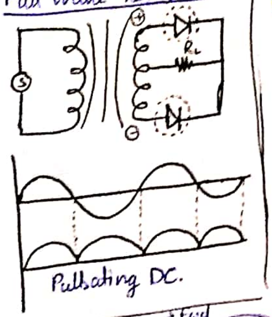
- It is converted into a constant DC by connecting a capacitor parallel to the load.



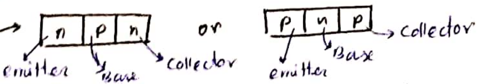
Fwd bias gives 0 resistance thus +ve part ~~is~~ is let through

Reversed bias prodives ∞ resistance i.e. no current.

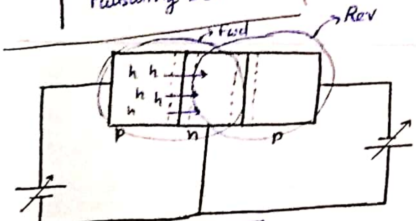
## Full Wave Rectifier:



## Transistor:



[illegible]:
- Emitter is thick and heavily doped
- Base is thin and lightly doped
- Collector is thickest and moderately doped.
- The purpose of emitter is to emitt majority carriers.
- Base provides an interface between the emitted & collector
- Collector is to collect majority carriers.

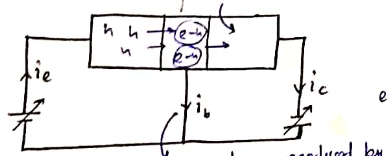


$$\boxed{i_e = i_b + i_c}$$

⇒

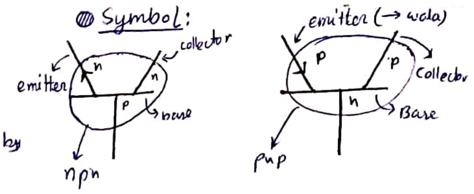
Only 5° (h-e) are formed

Rest 95% goes to −ve



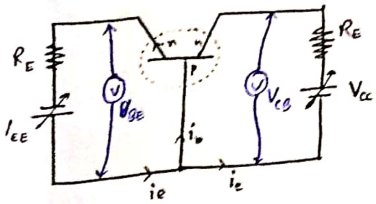
+ve charge produced by (h−e) formation.

**Symbol:**



## Common Base Mode: [Input is given thru emitter, output is taken thru collector]

$$\boxed{\beta_{DC} = \frac{i_c}{i_b}} \quad \boxed{R_{gain} = \frac{\Delta R_{out}}{\Delta R_{in}}}$$



$V_{EE}$ and $V_{cc}$ are of biasing battery

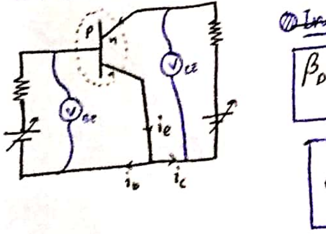
$V_{BE} \longrightarrow$ Input $V \neq V_{EE}$

$V_{CE} \longrightarrow$ Output $V \neq V_{cc}$

$$\boxed{\alpha_{AC} = \frac{\Delta i_c}{\Delta i_e}} \quad \boxed{V_{gain} = \alpha_{ac}}$$

$$\boxed{P_{gain} = \frac{\Delta i_c^2 R_{out}}{\Delta i_e^2 R_{in}} = \alpha^2_{ac} R_{gain}}$$

## Common Emitter Mode: [Input is given through base, Output is taken through emitter]



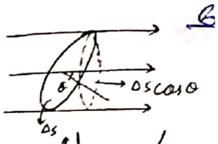
~~Input characteristics: (Gr~~

$$\boxed{\beta_{DC} = \frac{i_c}{i_b}} \quad \boxed{\beta_{AC} = \frac{\Delta i_c}{\Delta i_b}} \quad \boxed{R_{gain} = \frac{\Delta R_{out}}{\Delta R_{in}}} \quad \boxed{P_{gain} = \beta^2_{AC} R_{gain}} \quad \boxed{V_{gain} = \beta_{AC} R_{gain}}$$

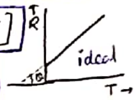
$$\boxed{\alpha = \frac{\beta}{1+\beta} \Leftrightarrow \frac{1}{\alpha} = 1 + \frac{1}{\beta}}$$

# CURRENT ELECTRICITY

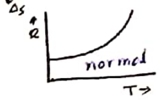
- **Electric Current:** $i = \frac{dQ}{dt}$
- **Current density:** $j = \frac{\Delta i}{\Delta S \cos\theta} \rightarrow \Delta i = j \Delta S \cos\theta \rightarrow \boxed{\Delta i = \hat{j} \cdot \Delta \vec{s} \quad i = \int \vec{j} \cdot d\vec{s}}$



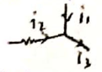
- **Relation of Drift speed & current density:** $\boxed{J = n v_d e}$, n → no. of electrons per unit volume.
- **Ohm's Law:** $\boxed{V = IR}$
- **Temp dependence of ρ (resistivity):** $\boxed{\rho(T) = \rho(T_0)[1 + \alpha \Delta T] \; , \; R = R_0[1 + \alpha \Delta T]}$



$\tan\theta = R_0 \alpha$

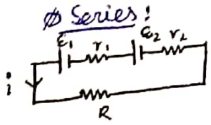


- **Kirchoff's Law:**
  - KCL: $i_3 = i_1 + i_2$



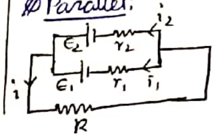
  - KVL: Loop.

- **Series combo (Resistor):** $\boxed{R_{eq} = \Sigma R_i}$
- **Parallel combo (Resistor):** $\boxed{\frac{1}{R_{eq}} = \Sigma \frac{1}{R_i}}$
- **Grouping of batteries:**
  - Series:



$i = \frac{E_1 + E_2}{R + (r_1 + r_2)}$

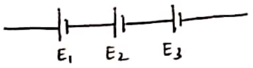
  - Parallel:



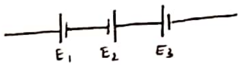
$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$

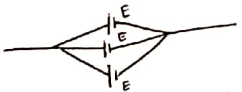
$i = \frac{E_{eq}}{r_{eq} + R}$



$E_{eq} = E_1 + E_2 + E_3$

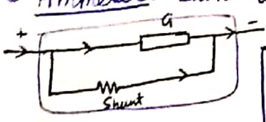


$E_{eq} = |E_1 - E_2 + E_3|$



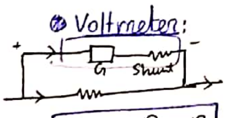
$E_{eq} = E$

- **Ammeter:** Shunt with low resistance in parallel.



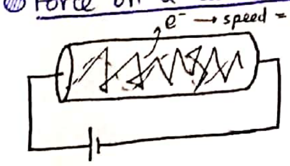
$\boxed{R_{eq} = \frac{R_G \cdot S}{R_G + S}}$

- **Voltmeter:**



$\boxed{R_{eq} = R_G + R}$

- **Force on a Curved Surface:**



concentration

$i = \frac{dq}{dt} = e \frac{dN}{dV} \cdot \frac{dV}{dt} = e \underset{\uparrow}{\eta} \times A \left(\frac{ds}{dt}\right)$

$\boxed{i = \eta e A V_d}$

- **Force on electron:** $\vec{F} = e\vec{E}$

$\boxed{\mu = \frac{e\tau}{m}}$ → avg relaxation time → electron mobility.

$\boxed{v_d = \frac{eE}{m}\tau}$ → Drift velocity.

$\boxed{\vec{j} = \sigma \vec{E}}$ → Conductivity.

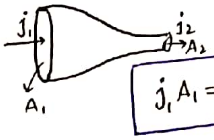
- **Resistivity:**

$\boxed{\rho = \frac{m}{\eta e^2 \tau}}$ depends on → Matter, Temperature.

$\boxed{T\uparrow \; \rho\uparrow \; R\uparrow}$

$\boxed{R = \rho \frac{l}{A}}$ → depends on Length, Area, Matter, Temp.

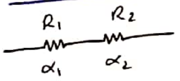
- **Equation of Continuity:**



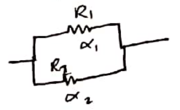
$\boxed{j_1 A_1 = j_2 A_2 = \cdots}$

$\boxed{RC = \rho \epsilon_0}$ → valid for all configuration.

- **α in series and parallel combo**

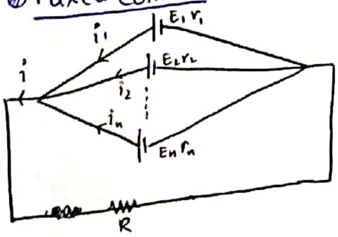


$\rightarrow \alpha_{eq}(s) = \frac{R_1 \alpha_1 + R_2 \alpha_2}{R_1 + R_2}$



$\rightarrow \alpha_{eq}(\parallel) = \frac{R_1 \alpha_2 + R_2 \alpha_1}{R_1 + R_2}$

- **Mixed Combos:**



$R_{eq} = R + \frac{1}{\Sigma 1/r_i}$

$E_1 - i_1 r_1 - iR = 0 \longrightarrow i_1 = \frac{E_1}{r_1} - \frac{iR}{r_1}$

$E_2 - i_2 r_2 - iR = 0 \longrightarrow i_2 = \frac{E_2}{r_2} - \frac{iR}{r_2}$

$\vdots$

$E_n - i_n r_n - iR = 0 \longrightarrow i_n = \frac{E_n}{r_n} - \frac{iR}{r_n}$

$i = \Sigma \frac{E}{r} - iR \Sigma \frac{1}{r}$

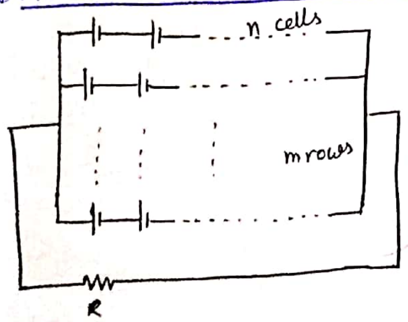
$i(1 + R\Sigma \frac{1}{r}) = \Sigma \frac{E}{r}$

$i = \frac{\Sigma E/r}{1 + R\Sigma 1/r}$

$\boxed{i = \frac{\Sigma E/r \Big/ \Sigma 1/r}{R + \frac{1}{\Sigma 1/r}}}$

$\boxed{E_{eq} = \frac{\Sigma E/r}{\Sigma 1/r}}$

- **Maximum Power Resistance Theorem:**



$i_{max}$ when,

$\boxed{R = \frac{nr}{m}}$

# THERMODYNAMICS

- **Work done:** $\boxed{W = -\int PdV = -P\Delta V}$
- **Work in Isothermal process:** $\boxed{W = 2.303nRT\log\left(\frac{V_f}{V_i}\right) = 2.303nRT\log\left(P_i/P_f\right)}$
- **First Law:** $\boxed{Q = \Delta U + W}$
- **Sign Convention:** $Q$ → + heat supplied to system; → − heat drawn from system

  $W$ → + work done by gas; → − work done on gas

  $\Delta U$ → + with temp rise; → − with temp fall
- **Heat:** $\boxed{Q = nC\Delta T}$

  Gram specific heat: $\boxed{C = \frac{Q}{m\Delta T}}$

  Molar specific heat: $\boxed{C = \frac{Q}{n\Delta T}}$
- **Special Cases:** → gas compressed suddenly, no heat supplied, ~~too~~ T rises.

  $\therefore C = \frac{Q}{n\Delta T} = 0$

  → gas is heated and allowed to expand so that T fall due to expantion = T rise due to heat, $\Delta T = 0$ $\therefore C = \infty$

  → gas ---- so that T fall ---- < T rise → $\Delta T > 0$ ∴ C ⊕ve

  → gas ---- T fall ---- > T rise — $\Delta T < 0$ ∴ C ⊖ve
- **$C_v$:** $\boxed{C_v = \frac{Q}{n\Delta T} \text{ (At V const)}}$

  ~~$Q = nC_v\Delta T$~~

  $\boxed{C_v = fR/2}$

  for any gas, $\boxed{\Delta U = nC_v\Delta T}$

  for isochoric, $\boxed{Q = \Delta U = nC_v\Delta T}$
- $\boxed{C_p - C_v = nR}$ $\boxed{C_p = C_v + R = \frac{fR}{2} + R = R\left(\frac{f}{2}+1\right)}$ $\boxed{\gamma = \frac{C_p}{C_v}}$
- $\boxed{C_v = \frac{R}{\gamma-1}}$ $\boxed{C_p = \frac{\gamma R}{\gamma-1}}$ $\boxed{U = nC_vT = \frac{nRT}{\gamma-1} = \frac{PV}{\gamma-1}}$
- **$C_p$:** $C_p = \frac{Q}{n\Delta T}$ (At P const)

  $\boxed{Q = nC_p\Delta T}$

$$\boxed{\Delta U = \frac{P_2V_2 - P_1V_1}{\gamma-1}}$$